Simple polytopes for three-dimensional isometry groups

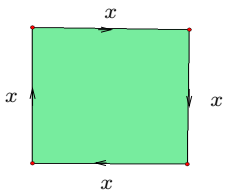

Figure 1.
$G = C_n$ $(n = 4)$, rotations of an n-prism with different coloured ends, generated by a rotation x.
$\mathrm{Dim}(P) = 2$. $G \cong \{x : x^n = 1\}$.

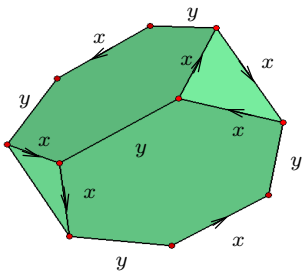

Figure 2.
$G = A_4$, rotations of a tetrahedron, generated by rotations x, y.
$\mathrm{Dim}(P) = 3$. $G \cong \{x, y : x^3 = y^2 = (xy)^3 = 1\}$.

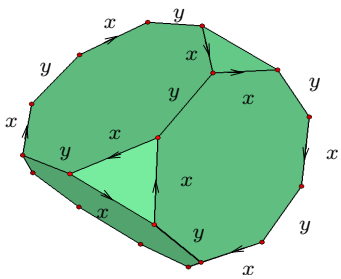

Figure 3.
$G = S_4$, rotations of a cube, generated by rotations x, y.
$\mathrm{Dim}(P) = 3$. $G \cong \{x, y : x^3 = y^2 = (xy)^4 = 1\}$.

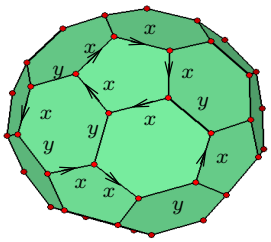

Figure 4.
$G = A_5$, rotations of an icosahedron, generated by rotations x, y.
$\mathrm{Dim}(P) = 3$. $G \cong \{x, y : x^5 = y^2 = (xy)^3 = 1\}$

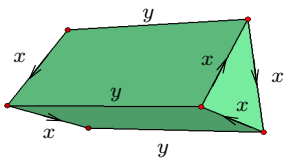

Figure 5.
$G = D_n$ $(n = 3)$, rotations of an n-prism, generated by rotations x, y.
$\mathrm{Dim}(P) = 3$. $G \cong \{x, y : x^n = y^2 = (xy)^2 = 1\}$.

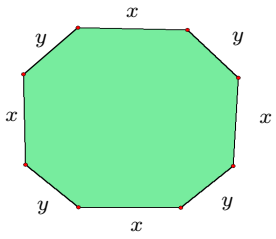

Figure 6.
$G = D_nC_n$ $(n = 4)$, symmetries of an n-prism with different coloured ends, generated by reflections x, y. $\mathrm{Dim}(P(G)) = 2$. $G \cong \{x, y : x^2 = y^2 = (xy)^n = 1\}$.

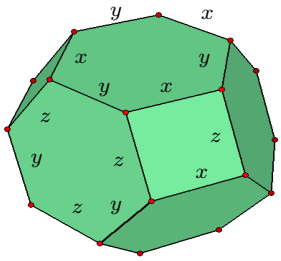

Figure 7.
$G = S_4A_4$, symmetries of a tetrahedron, generated by reflections x, y, z. $\mathrm{Dim}(P(G)) = 3$. $G \cong \{x, y, z : x^2 = y^2 = z^2 = (xy)^3 = (yz)^3 = (xz)^2 = 1\}$.

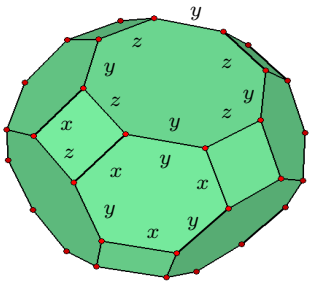

Figure 8.
$G = S_4 \times \langle J \rangle$, symmetries of a cube, generated by reflections x, y, z. $\mathrm{Dim}(P(G)) = 3$. $G \cong \{x, y, z : x^2 = y^2 = z^2 = (xy)^3 = (yz)^4 = (xz)^2 = 1\}$.

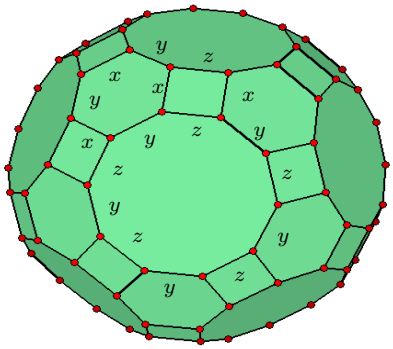

Figure 9.
$G = A_5 \times \langle J \rangle$, symmetries of an icosahedron, generated by reflections x, y, z. $\mathrm{Dim}(P(G)) = 3$. $G \cong \{x, y, z : x^2 = y^2 = z^2 = (xy)^3 = (yz)^5 = (xz)^2 = 1\}$.

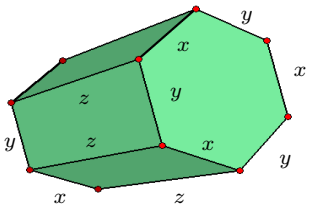

Figure 10.
$G = D_n \times \langle J \rangle$ $(n = 3)$, generated by reflections x, y, z.
$\mathrm{Dim}(P(G)) = 3$. $G \cong \{x, y, z : x^2 = y^2 = z^2 = (xy)^n = (xz)^2 = (yz)^2 = 1\}$.

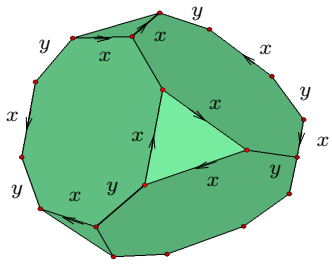

Figure 11.
$G = A_4 \times \langle J \rangle$, generated by a rotation x and reflection y.
$\mathrm{Dim}(P) = 3$. $G \cong \{x, y : x^3 = y^2 = (xyx^{-1}y)^2 = 1\}$.

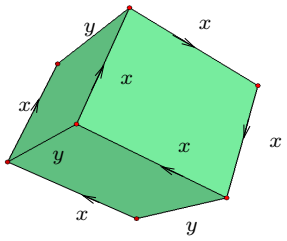

Figure 12.
$G = C_n \times \langle J \rangle$ $(n = 3)$, generated by a rotation x and reflection y.
$\mathrm{Dim}(P) = 3$. $G \cong \{x, y : x^n = y^2 = xyx^{-1}y = 1\}$