

# **Monoidal Categories**

## **Monoidal and monoidal (co)closed categories**

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# Chapter 1

## Monoidal Categories

### 1.1 Monoidal Categories

A 6-tuple  $(\mathbf{C}, \otimes, 1, \alpha, \lambda, \rho)$  consisting of

- a category  $\mathbf{C}$ ,
- a functor  $\otimes : \mathbf{C} \times \mathbf{C} \rightarrow \mathbf{C}$  compatible with the congruence of morphisms,
- an object  $1 \in \mathbf{C}$ ,
- a natural isomorphism  $\alpha_{a,b,c} : a \otimes (b \otimes c) \cong (a \otimes b) \otimes c$ ,
- a natural isomorphism  $\lambda_a : 1 \otimes a \cong a$ ,
- a natural isomorphism  $\rho_a : a \otimes 1 \cong a$ ,

is called a *monoidal category*, if

- for all objects  $a, b, c, d$ , the pentagon identity holds:

$$(\alpha_{a,b,c} \otimes \text{id}_d) \circ \alpha_{a,b \otimes c,d} \circ (\text{id}_a \otimes \alpha_{b,c,d}) \sim \alpha_{a \otimes b,c,d} \circ \alpha_{a,b,c \otimes d},$$

- for all objects  $a, c$ , the triangle identity holds:

$$(\rho_a \otimes \text{id}_c) \circ \alpha_{a,1,c} \sim \text{id}_a \otimes \lambda_c.$$

The corresponding GAP property is given by `IsMonoidalCategory`.

#### 1.1.1 TensorProductOnMorphisms (for IsCapCategoryMorphism, IsCapCategoryMorphism)

▷ `TensorProductOnMorphisms(alpha, beta)` (operation)

**Returns:** a morphism in  $\text{Hom}(a \otimes b, a' \otimes b')$

The arguments are two morphisms  $\alpha : a \rightarrow a', \beta : b \rightarrow b'$ . The output is the tensor product  $\alpha \otimes \beta$ .

### 1.1.2 TensorProductOnMorphismsWithGivenTensorProducts (for IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryMorphism, IsCapCategoryObject)

▷ TensorProductOnMorphismsWithGivenTensorProducts( $s$ ,  $\alpha$ ,  $\beta$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a \otimes b, a' \otimes b')$

The arguments are an object  $s = a \otimes b$ , two morphisms  $\alpha : a \rightarrow a'$ ,  $\beta : b \rightarrow b'$ , and an object  $r = a' \otimes b'$ . The output is the tensor product  $\alpha \otimes \beta$ .

### 1.1.3 AssociatorRightToLeft (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ AssociatorRightToLeft( $a$ ,  $b$ ,  $c$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a \otimes (b \otimes c), (a \otimes b) \otimes c)$ .

The arguments are three objects  $a, b, c$ . The output is the associator  $\alpha_{a,(b,c)} : a \otimes (b \otimes c) \rightarrow (a \otimes b) \otimes c$ .

### 1.1.4 AssociatorRightToLeftWithGivenTensorProducts (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ AssociatorRightToLeftWithGivenTensorProducts( $s$ ,  $a$ ,  $b$ ,  $c$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a \otimes (b \otimes c), (a \otimes b) \otimes c)$ .

The arguments are an object  $s = a \otimes (b \otimes c)$ , three objects  $a, b, c$ , and an object  $r = (a \otimes b) \otimes c$ . The output is the associator  $\alpha_{a,(b,c)} : a \otimes (b \otimes c) \rightarrow (a \otimes b) \otimes c$ .

### 1.1.5 AssociatorLeftToRight (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ AssociatorLeftToRight( $a$ ,  $b$ ,  $c$ ) (operation)

**Returns:** a morphism in  $\text{Hom}((a \otimes b) \otimes c \rightarrow a \otimes (b \otimes c))$ .

The arguments are three objects  $a, b, c$ . The output is the associator  $\alpha_{(a,b),c} : (a \otimes b) \otimes c \rightarrow a \otimes (b \otimes c)$ .

### 1.1.6 AssociatorLeftToRightWithGivenTensorProducts (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ AssociatorLeftToRightWithGivenTensorProducts( $s$ ,  $a$ ,  $b$ ,  $c$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}((a \otimes b) \otimes c \rightarrow a \otimes (b \otimes c))$ .

The arguments are an object  $s = (a \otimes b) \otimes c$ , three objects  $a, b, c$ , and an object  $r = a \otimes (b \otimes c)$ . The output is the associator  $\alpha_{(a,b),c} : (a \otimes b) \otimes c \rightarrow a \otimes (b \otimes c)$ .

### 1.1.7 LeftUnitor (for IsCapCategoryObject)

▷ LeftUnitor( $a$ ) (attribute)

**Returns:** a morphism in  $\text{Hom}(1 \otimes a, a)$

The argument is an object  $a$ . The output is the left unitor  $\lambda_a : 1 \otimes a \rightarrow a$ .

### 1.1.8 LeftUnitorWithGivenTensorProduct (for IsCapCategoryObject, IsCapCategoryObject)

▷ LeftUnitorWithGivenTensorProduct( $a, s$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(1 \otimes a, a)$

The arguments are an object  $a$  and an object  $s = 1 \otimes a$ . The output is the left unitor  $\lambda_a : 1 \otimes a \rightarrow a$ .

### 1.1.9 LeftUnitorInverse (for IsCapCategoryObject)

▷ LeftUnitorInverse( $a$ ) (attribute)

**Returns:** a morphism in  $\text{Hom}(a, 1 \otimes a)$

The argument is an object  $a$ . The output is the inverse of the left unitor  $\lambda_a^{-1} : a \rightarrow 1 \otimes a$ .

### 1.1.10 LeftUnitorInverseWithGivenTensorProduct (for IsCapCategoryObject, IsCapCategoryObject)

▷ LeftUnitorInverseWithGivenTensorProduct( $a, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a, 1 \otimes a)$

The argument is an object  $a$  and an object  $r = 1 \otimes a$ . The output is the inverse of the left unitor  $\lambda_a^{-1} : a \rightarrow 1 \otimes a$ .

### 1.1.11 RightUnitor (for IsCapCategoryObject)

▷ RightUnitor( $a$ ) (attribute)

**Returns:** a morphism in  $\text{Hom}(a \otimes 1, a)$

The argument is an object  $a$ . The output is the right unitor  $\rho_a : a \otimes 1 \rightarrow a$ .

### 1.1.12 RightUnitorWithGivenTensorProduct (for IsCapCategoryObject, IsCapCategoryObject)

▷ RightUnitorWithGivenTensorProduct( $a, s$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a \otimes 1, a)$

The arguments are an object  $a$  and an object  $s = a \otimes 1$ . The output is the right unitor  $\rho_a : a \otimes 1 \rightarrow a$ .

### 1.1.13 RightUnitorInverse (for IsCapCategoryObject)

▷ RightUnitorInverse( $a$ ) (attribute)

**Returns:** a morphism in  $\text{Hom}(a, a \otimes 1)$

The argument is an object  $a$ . The output is the inverse of the right unitor  $\rho_a^{-1} : a \rightarrow a \otimes 1$ .

### 1.1.14 RightUnitorInverseWithGivenTensorProduct (for IsCapCategoryObject, IsCapCategoryObject)

▷ RightUnitorInverseWithGivenTensorProduct( $a, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a, a \otimes 1)$

The arguments are an object  $a$  and an object  $r = a \otimes 1$ . The output is the inverse of the right unitor  $\rho_a^{-1} : a \rightarrow a \otimes 1$ .

### 1.1.15 TensorProductOnObjects (for IsCapCategoryObject, IsCapCategoryObject)

▷ `TensorProductOnObjects(a, b)` (operation)

**Returns:** an object

The arguments are two objects  $a, b$ . The output is the tensor product  $a \otimes b$ .

### 1.1.16 AddTensorProductOnObjects (for IsCapCategory, IsFunction)

▷ `AddTensorProductOnObjects(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `TensorProductOnObjects`.  $F : (a, b) \mapsto a \otimes b$ .

### 1.1.17 TensorUnit (for IsCapCategory)

▷ `TensorUnit(C)` (attribute)

**Returns:** an object

The argument is a category  $C$ . The output is the tensor unit  $1$  of  $C$ .

### 1.1.18 AddTensorUnit (for IsCapCategory, IsFunction)

▷ `AddTensorUnit(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `TensorUnit`.  $F : () \mapsto 1$ .

## 1.2 Additive Monoidal Categories

### 1.2.1 LeftDistributivityExpanding (for IsCapCategoryObject, IsList)

▷ `LeftDistributivityExpanding(a, L)` (operation)

**Returns:** a morphism in  $\text{Hom}(a \otimes (b_1 \oplus \dots \oplus b_n), (a \otimes b_1) \oplus \dots \oplus (a \otimes b_n))$

The arguments are an object  $a$  and a list of objects  $L = (b_1, \dots, b_n)$ . The output is the left distributivity morphism  $a \otimes (b_1 \oplus \dots \oplus b_n) \rightarrow (a \otimes b_1) \oplus \dots \oplus (a \otimes b_n)$ .

### 1.2.2 LeftDistributivityExpandingWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsList, IsCapCategoryObject)

▷ `LeftDistributivityExpandingWithGivenObjects(s, a, L, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$

The arguments are an object  $s = a \otimes (b_1 \oplus \dots \oplus b_n)$ , an object  $a$ , a list of objects  $L = (b_1, \dots, b_n)$ , and an object  $r = (a \otimes b_1) \oplus \dots \oplus (a \otimes b_n)$ . The output is the left distributivity morphism  $s \rightarrow r$ .

### 1.2.3 LeftDistributivityFactoring (for IsCapCategoryObject, IsList)

▷ `LeftDistributivityFactoring(a, L)` (operation)

**Returns:** a morphism in  $\text{Hom}((a \otimes b_1) \oplus \dots \oplus (a \otimes b_n), a \otimes (b_1 \oplus \dots \oplus b_n))$

The arguments are an object  $a$  and a list of objects  $L = (b_1, \dots, b_n)$ . The output is the left distributivity morphism  $(a \otimes b_1) \oplus \dots \oplus (a \otimes b_n) \rightarrow a \otimes (b_1 \oplus \dots \oplus b_n)$ .

#### 1.2.4 LeftDistributivityFactoringWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsList, IsCapCategoryObject)

▷ LeftDistributivityFactoringWithGivenObjects( $s, a, L, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$

The arguments are an object  $s = (a \otimes b_1) \oplus \dots \oplus (a \otimes b_n)$ , an object  $a$ , a list of objects  $L = (b_1, \dots, b_n)$ , and an object  $r = a \otimes (b_1 \oplus \dots \oplus b_n)$ . The output is the left distributivity morphism  $s \rightarrow r$ .

#### 1.2.5 RightDistributivityExpanding (for IsList, IsCapCategoryObject)

▷ RightDistributivityExpanding( $L, a$ ) (operation)

**Returns:** a morphism in  $\text{Hom}((b_1 \oplus \dots \oplus b_n) \otimes a, (b_1 \otimes a) \oplus \dots \oplus (b_n \otimes a))$

The arguments are a list of objects  $L = (b_1, \dots, b_n)$  and an object  $a$ . The output is the right distributivity morphism  $(b_1 \oplus \dots \oplus b_n) \otimes a \rightarrow (b_1 \otimes a) \oplus \dots \oplus (b_n \otimes a)$ .

#### 1.2.6 RightDistributivityExpandingWithGivenObjects (for IsCapCategoryObject, IsList, IsCapCategoryObject, IsCapCategoryObject)

▷ RightDistributivityExpandingWithGivenObjects( $s, L, a, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$

The arguments are an object  $s = (b_1 \oplus \dots \oplus b_n) \otimes a$ , a list of objects  $L = (b_1, \dots, b_n)$ , an object  $a$ , and an object  $r = (b_1 \otimes a) \oplus \dots \oplus (b_n \otimes a)$ . The output is the right distributivity morphism  $s \rightarrow r$ .

#### 1.2.7 RightDistributivityFactoring (for IsList, IsCapCategoryObject)

▷ RightDistributivityFactoring( $L, a$ ) (operation)

**Returns:** a morphism in  $\text{Hom}((b_1 \otimes a) \oplus \dots \oplus (b_n \otimes a), (b_1 \oplus \dots \oplus b_n) \otimes a)$

The arguments are a list of objects  $L = (b_1, \dots, b_n)$  and an object  $a$ . The output is the right distributivity morphism  $(b_1 \otimes a) \oplus \dots \oplus (b_n \otimes a) \rightarrow (b_1 \oplus \dots \oplus b_n) \otimes a$ .

#### 1.2.8 RightDistributivityFactoringWithGivenObjects (for IsCapCategoryObject, IsList, IsCapCategoryObject, IsCapCategoryObject)

▷ RightDistributivityFactoringWithGivenObjects( $s, L, a, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$

The arguments are an object  $s = (b_1 \otimes a) \oplus \dots \oplus (b_n \otimes a)$ , a list of objects  $L = (b_1, \dots, b_n)$ , an object  $a$ , and an object  $r = (b_1 \oplus \dots \oplus b_n) \otimes a$ . The output is the right distributivity morphism  $s \rightarrow r$ .

### 1.3 Braided Monoidal Categories

A monoidal category  $\mathcal{C}$  equipped with a natural isomorphism  $B_{a,b} : a \otimes b \cong b \otimes a$  is called a *braided monoidal category* if

- $\lambda_a \circ B_{a,1} \sim \rho_a$ ,



- $(B_{c,a} \otimes \text{id}_b) \circ \alpha_{c,a,b} \circ B_{a \otimes b, c} \sim \alpha_{a,c,b} \circ (\text{id}_a \otimes B_{b,c}) \circ \alpha_{a,b,c}^{-1}$ ,
- $(\text{id}_b \otimes B_{c,a}) \circ \alpha_{b,c,a}^{-1} \circ B_{a,b \otimes c} \sim \alpha_{b,a,c}^{-1} \circ (B_{a,b} \otimes \text{id}_c) \circ \alpha_{a,b,c}$ .

The corresponding GAP property is given by `IsBraidedMonoidalCategory`.

### 1.3.1 Braiding (for `IsCapCategoryObject`, `IsCapCategoryObject`)

▷ `Braiding(a, b)` (operation)

**Returns:** a morphism in  $\text{Hom}(a \otimes b, b \otimes a)$ .

The arguments are two objects  $a, b$ . The output is the braiding  $B_{a,b} : a \otimes b \rightarrow b \otimes a$ .

### 1.3.2 BraidingWithGivenTensorProducts (for `IsCapCategoryObject`, `IsCapCategoryObject`, `IsCapCategoryObject`)

▷ `BraidingWithGivenTensorProducts(s, a, b, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(a \otimes b, b \otimes a)$ .

The arguments are an object  $s = a \otimes b$ , two objects  $a, b$ , and an object  $r = b \otimes a$ . The output is the braiding  $B_{a,b} : a \otimes b \rightarrow b \otimes a$ .

### 1.3.3 BraidingInverse (for `IsCapCategoryObject`, `IsCapCategoryObject`)

▷ `BraidingInverse(a, b)` (operation)

**Returns:** a morphism in  $\text{Hom}(b \otimes a, a \otimes b)$ .

The arguments are two objects  $a, b$ . The output is the inverse braiding  $B_{a,b}^{-1} : b \otimes a \rightarrow a \otimes b$ .

### 1.3.4 BraidingInverseWithGivenTensorProducts (for `IsCapCategoryObject`, `IsCapCategoryObject`, `IsCapCategoryObject`)

▷ `BraidingInverseWithGivenTensorProducts(s, a, b, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(b \otimes a, a \otimes b)$ .

The arguments are an object  $s = b \otimes a$ , two objects  $a, b$ , and an object  $r = a \otimes b$ . The output is the inverse braiding  $B_{a,b}^{-1} : b \otimes a \rightarrow a \otimes b$ .

## 1.4 Symmetric Monoidal Categories

A braided monoidal category  $\mathbf{C}$  is called *symmetric monoidal category* if  $B_{a,b}^{-1} \sim B_{b,a}$ . The corresponding GAP property is given by `IsSymmetricMonoidalCategory`.

## 1.5 Closed Monoidal Categories

A monoidal category  $\mathbf{C}$  which has for each functor  $- \otimes b : \mathbf{C} \rightarrow \mathbf{C}$  a right adjoint (denoted by  $\underline{\text{Hom}}(b, -)$ ) is called a *closed monoidal category*.

If no operations involving duals are installed manually, the dual objects will be derived as  $a^\vee := \underline{\text{Hom}}(a, 1)$ .

The corresponding GAP property is called `IsClosedMonoidalCategory`.

### 1.5.1 InternalHomOnObjects (for IsCapCategoryObject, IsCapCategoryObject)

▷ `InternalHomOnObjects(a, b)` (operation)

**Returns:** an object

The arguments are two objects  $a, b$ . The output is the internal hom object  $\underline{\text{Hom}}(a, b)$ .

### 1.5.2 InternalHomOnMorphisms (for IsCapCategoryMorphism, IsCapCategoryMorphism)

▷ `InternalHomOnMorphisms(alpha, beta)` (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(a', b), \underline{\text{Hom}}(a, b'))$

The arguments are two morphisms  $\alpha : a \rightarrow a', \beta : b \rightarrow b'$ . The output is the internal hom morphism  $\underline{\text{Hom}}(\alpha, \beta) : \underline{\text{Hom}}(a', b) \rightarrow \underline{\text{Hom}}(a, b')$ .

### 1.5.3 InternalHomOnMorphismsWithGivenInternalHoms (for IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryMorphism, IsCapCategoryObject)

▷ `InternalHomOnMorphismsWithGivenInternalHoms(s, alpha, beta, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(a', b), \underline{\text{Hom}}(a, b'))$

The arguments are an object  $s = \underline{\text{Hom}}(a', b)$ , two morphisms  $\alpha : a \rightarrow a', \beta : b \rightarrow b'$ , and an object  $r = \underline{\text{Hom}}(a, b')$ . The output is the internal hom morphism  $\underline{\text{Hom}}(\alpha, \beta) : \underline{\text{Hom}}(a', b) \rightarrow \underline{\text{Hom}}(a, b')$ .

### 1.5.4 EvaluationMorphism (for IsCapCategoryObject, IsCapCategoryObject)

▷ `EvaluationMorphism(a, b)` (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(a, b) \otimes a, b)$ .

The arguments are two objects  $a, b$ . The output is the evaluation morphism  $\text{ev}_{a,b} : \underline{\text{Hom}}(a, b) \otimes a \rightarrow b$ , i.e., the counit of the tensor hom adjunction.

### 1.5.5 EvaluationMorphismWithGivenSource (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ `EvaluationMorphismWithGivenSource(a, b, s)` (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(a, b) \otimes a, b)$ .

The arguments are two objects  $a, b$  and an object  $s = \underline{\text{Hom}}(a, b) \otimes a$ . The output is the evaluation morphism  $\text{ev}_{a,b} : \underline{\text{Hom}}(a, b) \otimes a \rightarrow b$ , i.e., the counit of the tensor hom adjunction.

### 1.5.6 CoevaluationMorphism (for IsCapCategoryObject, IsCapCategoryObject)

▷ `CoevaluationMorphism(a, b)` (operation)

**Returns:** a morphism in  $\text{Hom}(a, \underline{\text{Hom}}(b, a \otimes b))$ .

The arguments are two objects  $a, b$ . The output is the coevaluation morphism  $\text{coev}_{a,b} : a \rightarrow \underline{\text{Hom}}(b, a \otimes b)$ , i.e., the unit of the tensor hom adjunction.

### 1.5.7 CoevaluationMorphismWithGivenRange (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ `CoevaluationMorphismWithGivenRange(a, b, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(a, \underline{\text{Hom}}(b, a \otimes b))$ .

The arguments are two objects  $a, b$  and an object  $r = \underline{\text{Hom}}(b, a \otimes b)$ . The output is the coevaluation morphism  $\text{coev}_{a,b} : a \rightarrow \underline{\text{Hom}}(b, a \otimes b)$ , i.e., the unit of the tensor hom adjunction.

### 1.5.8 TensorProductToInternalHomAdjunctionMap (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ `TensorProductToInternalHomAdjunctionMap(a, b, f)` (operation)

**Returns:** a morphism in  $\text{Hom}(a, \underline{\text{Hom}}(b, c))$ .

The arguments are two objects  $a, b$  and a morphism  $f : a \otimes b \rightarrow c$ . The output is a morphism  $g : a \rightarrow \underline{\text{Hom}}(b, c)$  corresponding to  $f$  under the tensor hom adjunction.

### 1.5.9 TensorProductToInternalHomAdjunctionMapWithGivenInternalHom (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryObject)

▷ `TensorProductToInternalHomAdjunctionMapWithGivenInternalHom(a, b, f, i)` (operation)

**Returns:** a morphism in  $\text{Hom}(a, \underline{\text{Hom}}(b, c))$ .

The arguments are two objects  $a, b$ , a morphism  $f : a \otimes b \rightarrow c$  and an object  $i = \underline{\text{Hom}}(b, c)$ . The output is a morphism  $g : a \rightarrow \underline{\text{Hom}}(b, c)$  corresponding to  $f$  under the tensor hom adjunction.

### 1.5.10 InternalHomToTensorProductAdjunctionMap (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ `InternalHomToTensorProductAdjunctionMap(b, c, g)` (operation)

**Returns:** a morphism in  $\text{Hom}(a \otimes b, c)$ .

The arguments are two objects  $b, c$  and a morphism  $g : a \rightarrow \underline{\text{Hom}}(b, c)$ . The output is a morphism  $f : a \otimes b \rightarrow c$  corresponding to  $g$  under the tensor hom adjunction.

### 1.5.11 InternalHomToTensorProductAdjunctionMapWithGivenTensorProduct (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryObject)

▷ `InternalHomToTensorProductAdjunctionMapWithGivenTensorProduct(b, c, g, t)` (operation)

**Returns:** a morphism in  $\text{Hom}(a \otimes b, c)$ .

The arguments are two objects  $b, c$ , a morphism  $g : a \rightarrow \underline{\text{Hom}}(b, c)$  and an object  $t = a \otimes b$ . The output is a morphism  $f : a \otimes b \rightarrow c$  corresponding to  $g$  under the tensor hom adjunction.

### 1.5.12 MonoidalPreComposeMorphism (for IsCapCategoryObject, IsCapCategory-Object, IsCapCategoryObject)

▷ MonoidalPreComposeMorphism( $a, b, c$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(a,b) \otimes \underline{\text{Hom}}(b,c), \underline{\text{Hom}}(a,c))$ .

The arguments are three objects  $a, b, c$ . The output is the precomposition morphism  $\text{MonoidalPreComposeMorphismWithGivenObjects}_{a,b,c} : \underline{\text{Hom}}(a,b) \otimes \underline{\text{Hom}}(b,c) \rightarrow \underline{\text{Hom}}(a,c)$ .

### 1.5.13 MonoidalPreComposeMorphismWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ MonoidalPreComposeMorphismWithGivenObjects( $s, a, b, c, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(a,b) \otimes \underline{\text{Hom}}(b,c), \underline{\text{Hom}}(a,c))$ .

The arguments are an object  $s = \underline{\text{Hom}}(a,b) \otimes \underline{\text{Hom}}(b,c)$ , three objects  $a, b, c$ , and an object  $r = \underline{\text{Hom}}(a,c)$ . The output is the precomposition morphism  $\text{MonoidalPreComposeMorphismWithGivenObjects}_{a,b,c} : \underline{\text{Hom}}(a,b) \otimes \underline{\text{Hom}}(b,c) \rightarrow \underline{\text{Hom}}(a,c)$ .

### 1.5.14 MonoidalPostComposeMorphism (for IsCapCategoryObject, IsCapCategory-Object, IsCapCategoryObject)

▷ MonoidalPostComposeMorphism( $a, b, c$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(b,c) \otimes \underline{\text{Hom}}(a,b), \underline{\text{Hom}}(a,c))$ .

The arguments are three objects  $a, b, c$ . The output is the postcomposition morphism  $\text{MonoidalPostComposeMorphismWithGivenObjects}_{a,b,c} : \underline{\text{Hom}}(b,c) \otimes \underline{\text{Hom}}(a,b) \rightarrow \underline{\text{Hom}}(a,c)$ .

### 1.5.15 MonoidalPostComposeMorphismWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ MonoidalPostComposeMorphismWithGivenObjects( $s, a, b, c, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(b,c) \otimes \underline{\text{Hom}}(a,b), \underline{\text{Hom}}(a,c))$ .

The arguments are an object  $s = \underline{\text{Hom}}(b,c) \otimes \underline{\text{Hom}}(a,b)$ , three objects  $a, b, c$ , and an object  $r = \underline{\text{Hom}}(a,c)$ . The output is the postcomposition morphism  $\text{MonoidalPostComposeMorphismWithGivenObjects}_{a,b,c} : \underline{\text{Hom}}(b,c) \otimes \underline{\text{Hom}}(a,b) \rightarrow \underline{\text{Hom}}(a,c)$ .

### 1.5.16 DualOnObjects (for IsCapCategoryObject)

▷ DualOnObjects( $a$ ) (attribute)

**Returns:** an object

The argument is an object  $a$ . The output is its dual object  $a^\vee$ .

### 1.5.17 DualOnMorphisms (for IsCapCategoryMorphism)

▷ DualOnMorphisms( $\alpha$ ) (attribute)

**Returns:** a morphism in  $\text{Hom}(b^\vee, a^\vee)$ .

The argument is a morphism  $\alpha : a \rightarrow b$ . The output is its dual morphism  $\alpha^\vee : b^\vee \rightarrow a^\vee$ .

### 1.5.18 DualOnMorphismsWithGivenDuals (for IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryObject)

▷ `DualOnMorphismsWithGivenDuals(s, alpha, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(b^\vee, a^\vee)$ .

The argument is an object  $s = b^\vee$ , a morphism  $\alpha : a \rightarrow b$ , and an object  $r = a^\vee$ . The output is the dual morphism  $\alpha^\vee : b^\vee \rightarrow a^\vee$ .

### 1.5.19 EvaluationForDual (for IsCapCategoryObject)

▷ `EvaluationForDual(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(a^\vee \otimes a, 1)$ .

The argument is an object  $a$ . The output is the evaluation morphism  $ev_a : a^\vee \otimes a \rightarrow 1$ .

### 1.5.20 EvaluationForDualWithGivenTensorProduct (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ `EvaluationForDualWithGivenTensorProduct(s, a, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(a^\vee \otimes a, 1)$ .

The arguments are an object  $s = a^\vee \otimes a$ , an object  $a$ , and an object  $r = 1$ . The output is the evaluation morphism  $ev_a : a^\vee \otimes a \rightarrow 1$ .

### 1.5.21 MorphismToBidual (for IsCapCategoryObject)

▷ `MorphismToBidual(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(a, (a^\vee)^\vee)$ .

The argument is an object  $a$ . The output is the morphism to the bidual  $a \rightarrow (a^\vee)^\vee$ .

### 1.5.22 MorphismToBidualWithGivenBidual (for IsCapCategoryObject, IsCapCategoryObject)

▷ `MorphismToBidualWithGivenBidual(a, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(a, (a^\vee)^\vee)$ .

The arguments are an object  $a$ , and an object  $r = (a^\vee)^\vee$ . The output is the morphism to the bidual  $a \rightarrow (a^\vee)^\vee$ .

### 1.5.23 TensorProductInternalHomCompatibilityMorphism (for IsList)

▷ `TensorProductInternalHomCompatibilityMorphism(list)` (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(a, a') \otimes \underline{\text{Hom}}(b, b'), \underline{\text{Hom}}(a \otimes b, a' \otimes b'))$ .

The argument is a list of four objects  $[a, a', b, b']$ . The output is the natural morphism `TensorProductInternalHomCompatibilityMorphismWithGivenObjectsa,a',b,b'` :  $\underline{\text{Hom}}(a, a') \otimes \underline{\text{Hom}}(b, b') \rightarrow \underline{\text{Hom}}(a \otimes b, a' \otimes b')$ .

### 1.5.24 TensorProductInternalHomCompatibilityMorphismWithGivenObjects (for IsCapCategoryObject, IsList, IsCapCategoryObject)

▷ TensorProductInternalHomCompatibilityMorphismWithGivenObjects( $s$ ,  $list$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(a, a') \otimes \underline{\text{Hom}}(b, b'), \underline{\text{Hom}}(a \otimes b, a' \otimes b'))$ .

The arguments are a list of four objects  $[a, a', b, b']$ , and two objects  $s = \underline{\text{Hom}}(a, a') \otimes \underline{\text{Hom}}(b, b')$  and  $r = \underline{\text{Hom}}(a \otimes b, a' \otimes b')$ . The output is the natural morphism  $\text{TensorProductInternalHomCompatibilityMorphismWithGivenObjects}_{a, a', b, b'} : \underline{\text{Hom}}(a, a') \otimes \underline{\text{Hom}}(b, b') \rightarrow \underline{\text{Hom}}(a \otimes b, a' \otimes b')$ .

### 1.5.25 TensorProductDualityCompatibilityMorphism (for IsCapCategoryObject, IsCapCategoryObject)

▷ TensorProductDualityCompatibilityMorphism( $a$ ,  $b$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a^\vee \otimes b^\vee, (a \otimes b)^\vee)$ .

The arguments are two objects  $a, b$ . The output is the natural morphism  $\text{TensorProductDualityCompatibilityMorphismWithGivenObjects} : a^\vee \otimes b^\vee \rightarrow (a \otimes b)^\vee$ .

### 1.5.26 TensorProductDualityCompatibilityMorphismWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ TensorProductDualityCompatibilityMorphismWithGivenObjects( $s$ ,  $a$ ,  $b$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a^\vee \otimes b^\vee, (a \otimes b)^\vee)$ .

The arguments are an object  $s = a^\vee \otimes b^\vee$ , two objects  $a, b$ , and an object  $r = (a \otimes b)^\vee$ . The output is the natural morphism  $\text{TensorProductDualityCompatibilityMorphismWithGivenObjects}_{a, b} : a^\vee \otimes b^\vee \rightarrow (a \otimes b)^\vee$ .

### 1.5.27 MorphismFromTensorProductToInternalHom (for IsCapCategoryObject, IsCapCategoryObject)

▷ MorphismFromTensorProductToInternalHom( $a$ ,  $b$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a^\vee \otimes b, \underline{\text{Hom}}(a, b))$ .

The arguments are two objects  $a, b$ . The output is the natural morphism  $\text{MorphismFromTensorProductToInternalHomWithGivenObjects}_{a, b} : a^\vee \otimes b \rightarrow \underline{\text{Hom}}(a, b)$ .

### 1.5.28 MorphismFromTensorProductToInternalHomWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ MorphismFromTensorProductToInternalHomWithGivenObjects( $s$ ,  $a$ ,  $b$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a^\vee \otimes b, \underline{\text{Hom}}(a, b))$ .

The arguments are an object  $s = a^\vee \otimes b$ , two objects  $a, b$ , and an object  $r = \underline{\text{Hom}}(a, b)$ . The output is the natural morphism  $\text{MorphismFromTensorProductToInternalHomWithGivenObjects}_{a, b} : a^\vee \otimes b \rightarrow \underline{\text{Hom}}(a, b)$ .

### 1.5.29 IsomorphismFromDualObjectToInternalHomIntoTensorUnit (for IsCapCategoryObject)

▷ `IsomorphismFromDualObjectToInternalHomIntoTensorUnit(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(a^\vee, \underline{\text{Hom}}(a, 1))$ .

The argument is an object  $a$ . The output is the isomorphism  $\text{IsomorphismFromDualObjectToInternalHomIntoTensorUnit}_a : a^\vee \rightarrow \underline{\text{Hom}}(a, 1)$ .

### 1.5.30 IsomorphismFromInternalHomIntoTensorUnitToDualObject (for IsCapCategoryObject)

▷ `IsomorphismFromInternalHomIntoTensorUnitToDualObject(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(a, 1), a^\vee)$ .

The argument is an object  $a$ . The output is the isomorphism  $\text{IsomorphismFromInternalHomIntoTensorUnitToDualObject}_a : \underline{\text{Hom}}(a, 1) \rightarrow a^\vee$ .

### 1.5.31 UniversalPropertyOfDual (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ `UniversalPropertyOfDual(t, a, alpha)` (operation)

**Returns:** a morphism in  $\text{Hom}(t, a^\vee)$ .

The arguments are two objects  $t, a$ , and a morphism  $\alpha : t \otimes a \rightarrow 1$ . The output is the morphism  $t \rightarrow a^\vee$  given by the universal property of  $a^\vee$ .

### 1.5.32 LambdaIntroduction (for IsCapCategoryMorphism)

▷ `LambdaIntroduction(alpha)` (attribute)

**Returns:** a morphism in  $\text{Hom}(1, \underline{\text{Hom}}(a, b))$ .

The argument is a morphism  $\alpha : a \rightarrow b$ . The output is the corresponding morphism  $1 \rightarrow \underline{\text{Hom}}(a, b)$  under the tensor hom adjunction.

### 1.5.33 LambdaElimination (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ `LambdaElimination(a, b, alpha)` (operation)

**Returns:** a morphism in  $\text{Hom}(a, b)$ .

The arguments are two objects  $a, b$ , and a morphism  $\alpha : 1 \rightarrow \underline{\text{Hom}}(a, b)$ . The output is a morphism  $a \rightarrow b$  corresponding to  $\alpha$  under the tensor hom adjunction.

### 1.5.34 IsomorphismFromObjectToInternalHom (for IsCapCategoryObject)

▷ `IsomorphismFromObjectToInternalHom(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(a, \underline{\text{Hom}}(1, a))$ .

The argument is an object  $a$ . The output is the natural isomorphism  $a \rightarrow \underline{\text{Hom}}(1, a)$ .

### 1.5.35 IsomorphismFromObjectToInternalHomWithGivenInternalHom (for IsCapCategoryObject, IsCapCategoryObject)

▷ `IsomorphismFromObjectToInternalHomWithGivenInternalHom(a, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(a, \underline{\text{Hom}}(1, a))$ .

The argument is an object  $a$ , and an object  $r = \underline{\text{Hom}}(1, a)$ . The output is the natural isomorphism  $a \rightarrow \underline{\text{Hom}}(1, a)$ .

### 1.5.36 IsomorphismFromInternalHomToObject (for IsCapCategoryObject)

▷ `IsomorphismFromInternalHomToObject(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(1, a), a)$ .

The argument is an object  $a$ . The output is the natural isomorphism  $\underline{\text{Hom}}(1, a) \rightarrow a$ .

### 1.5.37 IsomorphismFromInternalHomToObjectWithGivenInternalHom (for IsCapCategoryObject, IsCapCategoryObject)

▷ `IsomorphismFromInternalHomToObjectWithGivenInternalHom(a, s)` (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(1, a), a)$ .

The argument is an object  $a$ , and an object  $s = \underline{\text{Hom}}(1, a)$ . The output is the natural isomorphism  $\underline{\text{Hom}}(1, a) \rightarrow a$ .

## 1.6 Coclosed Monoidal Categories

A monoidal category  $\mathbf{C}$  which has for each functor  $- \otimes b : \mathbf{C} \rightarrow \mathbf{C}$  a left adjoint (denoted by  $\underline{\text{coHom}}(-, b)$ ) is called a *coclosed monoidal category*.

If no operations involving coduals are installed manually, the codual objects will be derived as  $a_\vee := \underline{\text{coHom}}(1, a)$ .

The corresponding GAP property is called `IsCoclosedMonoidalCategory`.

### 1.6.1 InternalCoHomOnObjects (for IsCapCategoryObject, IsCapCategoryObject)

▷ `InternalCoHomOnObjects(a, b)` (operation)

**Returns:** an object

The arguments are two objects  $a, b$ . The output is the internal cohom object  $\underline{\text{coHom}}(a, b)$ .

### 1.6.2 InternalCoHomOnMorphisms (for IsCapCategoryMorphism, IsCapCategoryMorphism)

▷ `InternalCoHomOnMorphisms(alpha, beta)` (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}(a, b'), \underline{\text{coHom}}(a', b))$

The arguments are two morphisms  $\alpha : a \rightarrow a', \beta : b \rightarrow b'$ . The output is the internal cohom morphism  $\underline{\text{coHom}}(\alpha, \beta) : \underline{\text{coHom}}(a, b') \rightarrow \underline{\text{coHom}}(a', b)$ .



### 1.6.3 InternalCoHomOnMorphismsWithGivenInternalCoHoms (for IsCapCategory-Object, IsCapCategoryMorphism, IsCapCategoryMorphism, IsCapCategory-Object)

▷ InternalCoHomOnMorphismsWithGivenInternalCoHoms( $s$ ,  $\alpha$ ,  $\beta$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}(a, b'), \underline{\text{coHom}}(a', b))$

The arguments are an object  $s = \underline{\text{coHom}}(a, b')$ , two morphisms  $\alpha : a \rightarrow a', \beta : b \rightarrow b'$ , and an object  $r = \underline{\text{coHom}}(a', b)$ . The output is the internal cohom morphism  $\underline{\text{coHom}}(\alpha, \beta) : \underline{\text{coHom}}(a, b') \rightarrow \underline{\text{coHom}}(a', b)$ .

### 1.6.4 CoclosedEvaluationMorphism (for IsCapCategoryObject, IsCapCategoryObject)

▷ CoclosedEvaluationMorphism( $a$ ,  $b$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a, \underline{\text{coHom}}(a, b) \otimes b)$ .

The arguments are two objects  $a, b$ . The output is the coclosed evaluation morphism  $\text{coclev}_{a,b} : a \rightarrow \underline{\text{coHom}}(a, b) \otimes b$ , i.e., the unit of the cohom tensor adjunction.

### 1.6.5 CoclosedEvaluationMorphismWithGivenRange (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ CoclosedEvaluationMorphismWithGivenRange( $a$ ,  $b$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a, \underline{\text{coHom}}(a, b) \otimes b)$ .

The arguments are two objects  $a, b$  and an object  $r = \underline{\text{coHom}}(a, b) \otimes b$ . The output is the coclosed evaluation morphism  $\text{coclev}_{a,b} : a \rightarrow \underline{\text{coHom}}(a, b) \otimes b$ , i.e., the unit of the cohom tensor adjunction.

### 1.6.6 CoclosedCoevaluationMorphism (for IsCapCategoryObject, IsCapCategoryObject)

▷ CoclosedCoevaluationMorphism( $a$ ,  $b$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}(a \otimes b, b), a)$ .

The arguments are two objects  $a, b$ . The output is the coclosed coevaluation morphism  $\text{coclcov}_{a,b} : \underline{\text{coHom}}(a \otimes b, b) \rightarrow a$ , i.e., the counit of the cohom tensor adjunction.

### 1.6.7 CoclosedCoevaluationMorphismWithGivenSource (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ CoclosedCoevaluationMorphismWithGivenSource( $a$ ,  $b$ ,  $s$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}(a \otimes b, b), b)$ .

The arguments are two objects  $a, b$  and an object  $s = \underline{\text{coHom}}(a \otimes b, b)$ . The output is the coclosed coevaluation morphism  $\text{coclcov}_{a,b} : \underline{\text{coHom}}(a \otimes b, b) \rightarrow a$ , i.e., the unit of the cohom tensor adjunction.

### 1.6.8 TensorProductToInternalCoHomAdjunctionMap (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ TensorProductToInternalCoHomAdjunctionMap( $c$ ,  $b$ ,  $g$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}(a, b), c)$ .

The arguments are two objects  $c, b$  and a morphism  $g : a \rightarrow c \otimes b$ . The output is a morphism  $f : \underline{\text{coHom}}(a, b) \rightarrow c$  corresponding to  $g$  under the cohom tensor adjunction.

### 1.6.9 TensorProductToInternalCoHomAdjunctionMapWithGivenInternalCoHom (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryObject)

▷ `TensorProductToInternalCoHomAdjunctionMapWithGivenInternalCoHom(c, b, g, i)` (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}(a, b), c)$ .

The arguments are two objects  $c, b$ , a morphism  $g : a \rightarrow c \otimes b$  and an object  $i = \underline{\text{coHom}}(a, b)$ . The output is a morphism  $f : \underline{\text{coHom}}(a, b) \rightarrow c$  corresponding to  $g$  under the cohom tensor adjunction.

### 1.6.10 InternalCoHomToTensorProductAdjunctionMap (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ `InternalCoHomToTensorProductAdjunctionMap(a, b, f)` (operation)

**Returns:** a morphism in  $\text{Hom}(a, c \otimes b)$ .

The arguments are two objects  $a, b$  and a morphism  $f : \underline{\text{coHom}}(a, b) \rightarrow c$ . The output is a morphism  $g : a \rightarrow c \otimes b$  corresponding to  $f$  under the cohom tensor adjunction.

### 1.6.11 InternalCoHomToTensorProductAdjunctionMapWithGivenTensorProduct (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryObject)

▷ `InternalCoHomToTensorProductAdjunctionMapWithGivenTensorProduct(a, b, f, t)` (operation)

**Returns:** a morphism in  $\text{Hom}(a, c \otimes b)$ .

The arguments are two objects  $a, b$ , a morphism  $f : \underline{\text{coHom}}(a, b) \rightarrow c$  and an object  $t = c \otimes b$ . The output is a morphism  $g : a \rightarrow c \otimes b$  corresponding to  $f$  under the cohom tensor adjunction.

### 1.6.12 MonoidalPreCoComposeMorphism (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ `MonoidalPreCoComposeMorphism(a, b, c)` (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}(a, c), \underline{\text{coHom}}(b, c) \otimes \underline{\text{coHom}}(a, b))$ .

The arguments are three objects  $a, b, c$ . The output is the precomposition morphism `MonoidalPreCoComposeMorphismWithGivenObjects` <sub>$a, b, c$</sub>  :  $\underline{\text{coHom}}(a, c) \rightarrow \underline{\text{coHom}}(b, c) \otimes \underline{\text{coHom}}(a, b)$ .

### 1.6.13 MonoidalPreCoComposeMorphismWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ `MonoidalPreCoComposeMorphismWithGivenObjects(s, a, b, c, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}(a, c), \underline{\text{coHom}}(b, c) \otimes \underline{\text{coHom}}(a, b))$ .

The arguments are an object  $s = \underline{\text{coHom}}(a, c)$ , three objects  $a, b, c$ , and an object  $r = \underline{\text{coHom}}(a, b) \otimes \underline{\text{coHom}}(b, c)$ . The output is the precomposition morphism

$\text{MonoidalPreCoComposeMorphismWithGivenObjects}_{a,b,c} : \underline{\text{coHom}}(a, c) \rightarrow \underline{\text{coHom}}(b, c) \otimes \underline{\text{coHom}}(a, b)$ .

#### 1.6.14 MonoidalPostCoComposeMorphism (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷  $\text{MonoidalPostCoComposeMorphism}(a, b, c)$  (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}(a, c), \underline{\text{coHom}}(a, b) \otimes \underline{\text{coHom}}(b, c))$ .

The arguments are three objects  $a, b, c$ . The output is the postcomposition morphism  $\text{MonoidalPostCoComposeMorphismWithGivenObjects}_{a,b,c} : \underline{\text{coHom}}(a, c) \rightarrow \underline{\text{coHom}}(a, b) \otimes \underline{\text{coHom}}(b, c)$ .

#### 1.6.15 MonoidalPostCoComposeMorphismWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷  $\text{MonoidalPostCoComposeMorphismWithGivenObjects}(s, a, b, c, r)$  (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}(a, c), \underline{\text{coHom}}(a, b) \otimes \underline{\text{coHom}}(b, c))$ .

The arguments are an object  $s = \underline{\text{coHom}}(a, c)$ , three objects  $a, b, c$ , and an object  $r = \underline{\text{coHom}}(b, c) \otimes \underline{\text{coHom}}(a, b)$ . The output is the postcomposition morphism  $\text{MonoidalPostCoComposeMorphismWithGivenObjects}_{a,b,c} : \underline{\text{coHom}}(a, c) \rightarrow \underline{\text{coHom}}(a, b) \otimes \underline{\text{coHom}}(b, c)$ .

#### 1.6.16 CoDualOnObjects (for IsCapCategoryObject)

▷  $\text{CoDualOnObjects}(a)$  (attribute)

**Returns:** an object

The argument is an object  $a$ . The output is its codual object  $a_\vee$ .

#### 1.6.17 CoDualOnMorphisms (for IsCapCategoryMorphism)

▷  $\text{CoDualOnMorphisms}(\alpha)$  (attribute)

**Returns:** a morphism in  $\text{Hom}(b_\vee, a_\vee)$ .

The argument is a morphism  $\alpha : a \rightarrow b$ . The output is its codual morphism  $\alpha_\vee : b_\vee \rightarrow a_\vee$ .

#### 1.6.18 CoDualOnMorphismsWithGivenCoDuals (for IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryObject)

▷  $\text{CoDualOnMorphismsWithGivenCoDuals}(s, \alpha, r)$  (operation)

**Returns:** a morphism in  $\text{Hom}(b_\vee, a_\vee)$ .

The argument is an object  $s = b_\vee$ , a morphism  $\alpha : a \rightarrow b$ , and an object  $r = a_\vee$ . The output is the dual morphism  $\alpha_\vee : b^\vee \rightarrow a^\vee$ .

#### 1.6.19 CoclosedEvaluationForCoDual (for IsCapCategoryObject)

▷  $\text{CoclosedEvaluationForCoDual}(a)$  (attribute)

**Returns:** a morphism in  $\text{Hom}(1, a_\vee \otimes a)$ .

The argument is an object  $a$ . The output is the coclosed evaluation morphism  $\text{coclev}_a : 1 \rightarrow a_\vee \otimes a$ .

### 1.6.20 CoclosedEvaluationForCoDualWithGivenTensorProduct (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ `CoclosedEvaluationForCoDualWithGivenTensorProduct(s, a, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(1, a_{\vee} \otimes a)$ .

The arguments are an object  $s = 1$ , an object  $a$ , and an object  $r = a_{\vee} \otimes a$ . The output is the coclosed evaluation morphism  $\text{coclev}_a : 1 \rightarrow a_{\vee} \otimes a$ .

### 1.6.21 MorphismFromCoBidual (for IsCapCategoryObject)

▷ `MorphismFromCoBidual(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}((a_{\vee})_{\vee}, a)$ .

The argument is an object  $a$ . The output is the morphism from the cobidual  $(a_{\vee})_{\vee} \rightarrow a$ .

### 1.6.22 MorphismFromCoBidualWithGivenCoBidual (for IsCapCategoryObject, IsCapCategoryObject)

▷ `MorphismFromCoBidualWithGivenCoBidual(a, s)` (operation)

**Returns:** a morphism in  $\text{Hom}((a_{\vee})_{\vee}, a)$ .

The arguments are an object  $a$ , and an object  $s = (a_{\vee})_{\vee}$ . The output is the morphism from the cobidual  $(a_{\vee})_{\vee} \rightarrow a$ .

### 1.6.23 InternalCoHomTensorProductCompatibilityMorphism (for IsList)

▷ `InternalCoHomTensorProductCompatibilityMorphism(list)` (operation)

**Returns:** a morphism in  $\text{Hom}(\text{coHom}(a \otimes a', b \otimes b'), \text{coHom}(a, b) \otimes \text{coHom}(a', b'))$ .

The argument is a list of four objects  $[a, a', b, b']$ . The output is the natural morphism  $\text{InternalCoHomTensorProductCompatibilityMorphismWithGivenObjects}_{a, a', b, b'} : \text{coHom}(a \otimes a', b \otimes b') \rightarrow \text{coHom}(a, b) \otimes \text{coHom}(a', b')$ .

### 1.6.24 InternalCoHomTensorProductCompatibilityMorphismWithGivenObjects (for IsCapCategoryObject, IsList, IsCapCategoryObject)

▷ `InternalCoHomTensorProductCompatibilityMorphismWithGivenObjects(s, list, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(\text{coHom}(a \otimes a', b \otimes b'), \text{coHom}(a, b) \otimes \text{coHom}(a', b'))$ .

The arguments are a list of four objects  $[a, a', b, b']$ , and two objects  $s = \text{coHom}(a \otimes a', b \otimes b')$  and  $r = \text{coHom}(a, b) \otimes \text{coHom}(a', b')$ . The output is the natural morphism  $\text{InternalCoHomTensorProductCompatibilityMorphismWithGivenObjects}_{a, a', b, b'} : \text{coHom}(a \otimes a', b \otimes b') \rightarrow \text{coHom}(a, b) \otimes \text{coHom}(a', b')$ .

### 1.6.25 CoDualityTensorProductCompatibilityMorphism (for IsCapCategoryObject, IsCapCategoryObject)

▷ `CoDualityTensorProductCompatibilityMorphism(a, b)` (operation)

**Returns:** a morphism in  $\text{Hom}((a \otimes b)_{\vee}, a_{\vee} \otimes b_{\vee})$ .

The arguments are two objects  $a, b$ . The output is the natural morphism  $\text{CoDualityTensorProductCompatibilityMorphismWithGivenObjects} : (a \otimes b)_{\vee} \rightarrow a_{\vee} \otimes b_{\vee}$ .

### 1.6.26 CoDualityTensorProductCompatibilityMorphismWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ CoDualityTensorProductCompatibilityMorphismWithGivenObjects( $s, a, b, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}((a \otimes b)_V, a_V \otimes b_V)$ .

The arguments are an object  $s = (a \otimes b)_V$ , two objects  $a, b$ , and an object  $r = a_V \otimes b_V$ . The output is the natural morphism  $\text{CoDualityTensorProductCompatibilityMorphismWithGivenObjects}_{a,b} : (a \otimes b)_V \rightarrow a_V \otimes b_V$ .

### 1.6.27 MorphismFromInternalCoHomToTensorProduct (for IsCapCategoryObject, IsCapCategoryObject)

▷ MorphismFromInternalCoHomToTensorProduct( $a, b$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\text{coHom}(a, b), b_V \otimes a)$ .

The arguments are two objects  $a, b$ . The output is the natural morphism  $\text{MorphismFromInternalCoHomToTensorProductWithGivenObjects}_{a,b} : \text{coHom}(a, b) \rightarrow b_V \otimes a$ .

### 1.6.28 MorphismFromInternalCoHomToTensorProductWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ MorphismFromInternalCoHomToTensorProductWithGivenObjects( $s, a, b, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\text{coHom}(a, b), a \otimes b_V)$ .

The arguments are an object  $s = \text{coHom}(a, b)$ , two objects  $a, b$ , and an object  $r = b_V \otimes a$ . The output is the natural morphism  $\text{MorphismFromInternalCoHomToTensorProductWithGivenObjects}_{a,b} : \text{coHom}(a, b) \rightarrow a \otimes b_V$ .

### 1.6.29 IsomorphismFromCoDualObjectToInternalCoHomFromTensorUnit (for IsCapCategoryObject)

▷ IsomorphismFromCoDualObjectToInternalCoHomFromTensorUnit( $a$ ) (attribute)

**Returns:** a morphism in  $\text{Hom}(a_V, \text{coHom}(1, a))$ .

The argument is an object  $a$ . The output is the isomorphism  $\text{IsomorphismFromCoDualObjectToInternalCoHomFromTensorUnit}_a : a_V \rightarrow \text{coHom}(1, a)$ .

### 1.6.30 IsomorphismFromInternalCoHomFromTensorUnitToCoDualObject (for IsCapCategoryObject)

▷ IsomorphismFromInternalCoHomFromTensorUnitToCoDualObject( $a$ ) (attribute)

**Returns:** a morphism in  $\text{Hom}(\text{coHom}(1, a), a_V)$ .

The argument is an object  $a$ . The output is the isomorphism  $\text{IsomorphismFromInternalCoHomFromTensorUnitToCoDualObject}_a : \text{coHom}(1, a) \rightarrow a_V$ .

### 1.6.31 UniversalPropertyOfCoDual (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ `UniversalPropertyOfCoDual( $t$ ,  $a$ ,  $alpha$ )` (operation)

**Returns:** a morphism in  $\text{Hom}(a_{\vee}, t)$ .

The arguments are two objects  $t, a$ , and a morphism  $\alpha : 1 \rightarrow t \otimes a$ . The output is the morphism  $a_{\vee} \rightarrow t$  given by the universal property of  $a_{\vee}$ .

### 1.6.32 CoLambdaIntroduction (for IsCapCategoryMorphism)

▷ `CoLambdaIntroduction( $alpha$ )` (attribute)

**Returns:** a morphism in  $\text{Hom}(\text{coHom}(a, b), 1)$ .

The argument is a morphism  $\alpha : a \rightarrow b$ . The output is the corresponding morphism  $\text{coHom}(a, b) \rightarrow 1$  under the cohom tensor adjunction.

### 1.6.33 CoLambdaElimination (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ `CoLambdaElimination( $a$ ,  $b$ ,  $alpha$ )` (operation)

**Returns:** a morphism in  $\text{Hom}(a, b)$ .

The arguments are two objects  $a, b$ , and a morphism  $\alpha : \text{coHom}(a, b) \rightarrow 1$ . The output is a morphism  $a \rightarrow b$  corresponding to  $\alpha$  under the cohom tensor adjunction.

### 1.6.34 IsomorphismFromObjectToInternalCoHom (for IsCapCategoryObject)

▷ `IsomorphismFromObjectToInternalCoHom( $a$ )` (attribute)

**Returns:** a morphism in  $\text{Hom}(a, \text{coHom}(a, 1))$ .

The argument is an object  $a$ . The output is the natural isomorphism  $a \rightarrow \text{coHom}(a, 1)$ .

### 1.6.35 IsomorphismFromObjectToInternalCoHomWithGivenInternalCoHom (for IsCapCategoryObject, IsCapCategoryObject)

▷ `IsomorphismFromObjectToInternalCoHomWithGivenInternalCoHom( $a$ ,  $r$ )` (operation)

**Returns:** a morphism in  $\text{Hom}(a, \text{coHom}(a, 1))$ .

The argument is an object  $a$ , and an object  $r = \text{coHom}(a, 1)$ . The output is the natural isomorphism  $a \rightarrow \text{coHom}(a, 1)$ .

### 1.6.36 IsomorphismFromInternalCoHomToObject (for IsCapCategoryObject)

▷ `IsomorphismFromInternalCoHomToObject( $a$ )` (attribute)

**Returns:** a morphism in  $\text{Hom}(\text{coHom}(a, 1), a)$ .

The argument is an object  $a$ . The output is the natural isomorphism  $\text{coHom}(a, 1) \rightarrow a$ .

### 1.6.37 IsomorphismFromInternalCoHomToObjectWithGivenInternalCoHom (for IsCapCategoryObject, IsCapCategoryObject)

▷ `IsomorphismFromInternalCoHomToObjectWithGivenInternalCoHom( $a$ ,  $s$ )` (operation)

**Returns:** a morphism in  $\text{Hom}(\text{coHom}(a, 1), a)$ .

The argument is an object  $a$ , and an object  $s = \text{coHom}(a, 1)$ . The output is the natural isomorphism  $\text{coHom}(a, 1) \rightarrow a$ .

## 1.7 Symmetric Closed Monoidal Categories

A monoidal category  $\mathbf{C}$  which is symmetric and closed is called a *symmetric closed monoidal category*. The corresponding GAP property is given by `IsSymmetricClosedMonoidalCategory`.

## 1.8 Symmetric Coclosed Monoidal Categories

A monoidal category  $\mathbf{C}$  which is symmetric and coclosed is called a *symmetric coclosed monoidal category*.

The corresponding GAP property is given by `IsSymmetricCoclosedMonoidalCategory`.

## 1.9 Rigid Symmetric Closed Monoidal Categories

A symmetric closed monoidal category  $\mathbf{C}$  satisfying

- the natural morphism

$\text{Hom}(a, a') \otimes \text{Hom}(b, b') \rightarrow \text{Hom}(a \otimes b, a' \otimes b')$  is an isomorphism,

- the natural morphism

$a \rightarrow \text{Hom}(\text{Hom}(a, 1), 1)$  is an isomorphism is called a *rigid symmetric closed monoidal category*.

If no operations involving the closed structure are installed manually, the internal hom objects will be derived as  $\text{Hom}(a, b) := a^\vee \otimes b$  and, in particular,  $\text{Hom}(a, 1) := a^\vee \otimes 1$ .

The corresponding GAP property is given by `IsRigidSymmetricClosedMonoidalCategory`.

### 1.9.1 IsomorphismFromTensorProductWithDualObjectToInternalHom (for IsCapCategoryObject, IsCapCategoryObject)

▷ `IsomorphismFromTensorProductWithDualObjectToInternalHom(a, b)` (operation)

**Returns:** a morphism in  $\text{Hom}(a^\vee \otimes b, \text{Hom}(a, b))$ .

The arguments are two objects  $a, b$ . The output is the natural morphism  $\text{IsomorphismFromTensorProductWithDualObjectToInternalHom}_{a,b} : a^\vee \otimes b \rightarrow \text{Hom}(a, b)$ .

### 1.9.2 IsomorphismFromInternalHomToTensorProductWithDualObject (for IsCapCategoryObject, IsCapCategoryObject)

▷ `IsomorphismFromInternalHomToTensorProductWithDualObject(a, b)` (operation)

**Returns:** a morphism in  $\text{Hom}(\text{Hom}(a, b), a^\vee \otimes b)$ .

The arguments are two objects  $a, b$ . The output is the inverse of  $\text{IsomorphismFromTensorProductWithDualObjectToInternalHom}$ , namely  $\text{IsomorphismFromInternalHomToTensorProductWithDualObject}_{a,b} : \text{Hom}(a, b) \rightarrow a^\vee \otimes b$ .

### 1.9.3 MorphismFromInternalHomToTensorProduct (for IsCapCategoryObject, Is-CapCategoryObject)

▷ MorphismFromInternalHomToTensorProduct( $a, b$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(a, b), a^\vee \otimes b)$ .

The arguments are two objects  $a, b$ . The output is the inverse of MorphismFromTensorProductToInternalHomWithGivenObjects, namely MorphismFromInternalHomToTensorProductWithGivenObjects $_{a,b} : \underline{\text{Hom}}(a, b) \rightarrow a^\vee \otimes b$ .

### 1.9.4 MorphismFromInternalHomToTensorProductWithGivenObjects (for IsCap-CategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategory-Object)

▷ MorphismFromInternalHomToTensorProductWithGivenObjects( $s, a, b, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(a, b), a^\vee \otimes b)$ .

The arguments are an object  $s = \underline{\text{Hom}}(a, b)$ , two objects  $a, b$ , and an object  $r = a^\vee \otimes b$ . The output is the inverse of MorphismFromTensorProductToInternalHomWithGivenObjects, namely MorphismFromInternalHomToTensorProductWithGivenObjects $_{a,b} : \underline{\text{Hom}}(a, b) \rightarrow a^\vee \otimes b$ .

### 1.9.5 TensorProductInternalHomCompatibilityMorphismInverse (for IsList)

▷ TensorProductInternalHomCompatibilityMorphismInverse( $list$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(a \otimes b, a' \otimes b'), \underline{\text{Hom}}(a, a') \otimes \underline{\text{Hom}}(b, b'))$ .

The argument is a list of four objects  $[a, a', b, b']$ . The output is the natural morphism TensorProductInternalHomCompatibilityMorphismInverseWithGivenObjects $_{a,a',b,b'} : \underline{\text{Hom}}(a \otimes b, a' \otimes b') \rightarrow \underline{\text{Hom}}(a, a') \otimes \underline{\text{Hom}}(b, b')$ .

### 1.9.6 TensorProductInternalHomCompatibilityMorphismInverseWithGivenObjects (for IsCapCategoryObject, IsList, IsCapCategoryObject)

▷ TensorProductInternalHomCompatibilityMorphismInverseWithGivenObjects( $s, list, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(a \otimes b, a' \otimes b'), \underline{\text{Hom}}(a, a') \otimes \underline{\text{Hom}}(b, b'))$ .

The arguments are a list of four objects  $[a, a', b, b']$ , and two objects  $s = \underline{\text{Hom}}(a \otimes b, a' \otimes b')$  and  $r = \underline{\text{Hom}}(a, a') \otimes \underline{\text{Hom}}(b, b')$ . The output is the natural morphism TensorProductInternalHomCompatibilityMorphismInverseWithGivenObjects $_{a,a',b,b'} : \underline{\text{Hom}}(a \otimes b, a' \otimes b') \rightarrow \underline{\text{Hom}}(a, a') \otimes \underline{\text{Hom}}(b, b')$ .

### 1.9.7 CoevaluationForDual (for IsCapCategoryObject)

▷ CoevaluationForDual( $a$ ) (attribute)

**Returns:** a morphism in  $\text{Hom}(1, a \otimes a^\vee)$ .

The argument is an object  $a$ . The output is the coevaluation morphism  $\text{coev}_a : 1 \rightarrow a \otimes a^\vee$ .



### 1.9.8 CoevaluationForDualWithGivenTensorProduct (for IsCapCategoryObject, Is-CapCategoryObject, IsCapCategoryObject)

▷ `CoevaluationForDualWithGivenTensorProduct(s, a, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(1, a \otimes a^\vee)$ .

The arguments are an object  $s = 1$ , an object  $a$ , and an object  $r = a \otimes a^\vee$ . The output is the coevaluation morphism  $\text{coev}_a : 1 \rightarrow a \otimes a^\vee$ .

### 1.9.9 TraceMap (for IsCapCategoryMorphism)

▷ `TraceMap(alpha)` (attribute)

**Returns:** a morphism in  $\text{Hom}(1, 1)$ .

The argument is an endomorphism  $\alpha : a \rightarrow a$ . The output is the trace morphism  $\text{trace}_\alpha : 1 \rightarrow 1$ .

### 1.9.10 RankMorphism (for IsCapCategoryObject)

▷ `RankMorphism(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(1, 1)$ .

The argument is an object  $a$ . The output is the rank morphism  $\text{rank}_a : 1 \rightarrow 1$ .

### 1.9.11 MorphismFromBidual (for IsCapCategoryObject)

▷ `MorphismFromBidual(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}((a^\vee)^\vee, a)$ .

The argument is an object  $a$ . The output is the inverse of the morphism to the bidual  $(a^\vee)^\vee \rightarrow a$ .

### 1.9.12 MorphismFromBidualWithGivenBidual (for IsCapCategoryObject, IsCapCategoryObject)

▷ `MorphismFromBidualWithGivenBidual(a, s)` (operation)

**Returns:** a morphism in  $\text{Hom}((a^\vee)^\vee, a)$ .

The argument is an object  $a$ , and an object  $s = (a^\vee)^\vee$ . The output is the inverse of the morphism to the bidual  $(a^\vee)^\vee \rightarrow a$ .

## 1.10 Rigid Symmetric Coclosed Monoidal Categories

A symmetric coclosed monoidal category  $\mathbf{C}$  satisfying

- the natural morphism

$\underline{\text{coHom}}(a \otimes a', b \otimes b') \rightarrow \underline{\text{coHom}}(a, b) \otimes \underline{\text{coHom}}(a', b')$  is an isomorphism,

- the natural morphism

$\underline{\text{coHom}}(1, \underline{\text{coHom}}(1, a)) \rightarrow a$  is an isomorphism is called a *rigid symmetric coclosed monoidal category*.

If no operations involving the coclosed structure are installed manually, the internal cohom objects will be derived as  $\underline{\text{coHom}}(a, b) := a \otimes b_\vee$  and, in particular,  $\underline{\text{coHom}}(1, a) := 1 \otimes a_\vee$ .

The corresponding GAP property is given by `IsRigidSymmetricCoclosedMonoidalCategory`.

### 1.10.1 IsomorphismFromInternalCoHomToTensorProductWithCoDualObject (for IsCapCategoryObject, IsCapCategoryObject)

▷ `IsomorphismFromInternalCoHomToTensorProductWithCoDualObject(a, b)` (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}(a, b), b_{\vee} \otimes a)$ .

The arguments are two objects  $a, b$ . The output is the natural morphism `IsomorphismFromInternalCoHomToTensorProductWithCoDualObjectWithGivenObjectsa,b` :  $\underline{\text{coHom}}(a, b) \rightarrow b_{\vee} \otimes a$ .

### 1.10.2 IsomorphismFromTensorProductWithCoDualObjectToInternalCoHom (for IsCapCategoryObject, IsCapCategoryObject)

▷ `IsomorphismFromTensorProductWithCoDualObjectToInternalCoHom(a, b)` (operation)

**Returns:** a morphism in  $\text{Hom}(a_{\vee} \otimes b, \underline{\text{coHom}}(b, a))$ .

The arguments are two objects  $a, b$ . The output is the inverse of `IsomorphismFromInternalCoHomToTensorProductWithCoDualObject`, namely `IsomorphismFromTensorProductWithCoDualObjectToInternalCoHoma,b` :  $a_{\vee} \otimes b \rightarrow \underline{\text{coHom}}(b, a)$ .

### 1.10.3 MorphismFromTensorProductToInternalCoHom (for IsCapCategoryObject, IsCapCategoryObject)

▷ `MorphismFromTensorProductToInternalCoHom(a, b)` (operation)

**Returns:** a morphism in  $\text{Hom}(a_{\vee} \otimes b, \underline{\text{coHom}}(b, a))$ .

The arguments are two objects  $a, b$ . The output is the inverse of `MorphismFromInternalCoHomToTensorProductWithGivenObjects`, namely `MorphismFromTensorProductToInternalCoHomWithGivenObjectsa,b` :  $a_{\vee} \otimes b \rightarrow \underline{\text{coHom}}(b, a)$ .

### 1.10.4 MorphismFromTensorProductToInternalCoHomWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ `MorphismFromTensorProductToInternalCoHomWithGivenObjects(s, a, b, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(a_{\vee} \otimes b, \underline{\text{coHom}}(b, a))$ .

The arguments are an object  $s_{\vee} = a \otimes b$ , two objects  $a, b$ , and an object  $r = \underline{\text{coHom}}(b, a)$ . The output is the inverse of `MorphismFromInternalCoHomToTensorProductWithGivenObjects`, namely `MorphismFromTensorProductToInternalCoHomWithGivenObjectsa,b` :  $a_{\vee} \otimes b \rightarrow \underline{\text{coHom}}(b, a)$ .

### 1.10.5 InternalCoHomTensorProductCompatibilityMorphismInverse (for IsList)

▷ `InternalCoHomTensorProductCompatibilityMorphismInverse(list)` (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}(a, b) \otimes \underline{\text{coHom}}(a', b'), \underline{\text{coHom}}(a \otimes a', b \otimes b'))$ .

The argument is a list of four objects  $[a, a', b, b']$ . The output is the natural morphism `InternalCoHomTensorProductCompatibilityMorphismInverseWithGivenObjectsa,a',b,b'` :  $\underline{\text{coHom}}(a, b) \otimes \underline{\text{coHom}}(a', b') \rightarrow \underline{\text{coHom}}(a \otimes a', b \otimes b')$ .

### 1.10.6 InternalCoHomTensorProductCompatibilityMorphismInverseWithGivenObjects (for IsCapCategoryObject, IsList, IsCapCategoryObject)

▷ InternalCoHomTensorProductCompatibilityMorphismInverseWithGivenObjects( $s$ ,  $list$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}(a, b) \otimes \underline{\text{coHom}}(a', b'), \underline{\text{coHom}}(a \otimes a', b \otimes b'))$ .

The arguments are a list of four objects  $[a, a', b, b']$ , and two objects  $s = \underline{\text{coHom}}(a, b) \otimes \underline{\text{coHom}}(a', b')$  and  $r = \underline{\text{coHom}}(a \otimes a', b \otimes b')$ . The output is the natural morphism  $\text{InternalCoHomTensorProductCompatibilityMorphismInverseWithGivenObjects}_{a, a', b, b'} : \underline{\text{coHom}}(a, b) \otimes \underline{\text{coHom}}(a', b') \rightarrow \underline{\text{coHom}}(a \otimes a', b \otimes b')$ .

### 1.10.7 CoclosedCoevaluationForCoDual (for IsCapCategoryObject)

▷ CoclosedCoevaluationForCoDual( $a$ ) (attribute)

**Returns:** a morphism in  $\text{Hom}(a \otimes a_V, 1)$ .

The argument is an object  $a$ . The output is the coclosed coevaluation morphism  $\text{coclcov}_a : a \otimes a_V \rightarrow 1$ .

### 1.10.8 CoclosedCoevaluationForCoDualWithGivenTensorProduct (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ CoclosedCoevaluationForCoDualWithGivenTensorProduct( $s$ ,  $a$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a \otimes a_V, 1)$ .

The arguments are an object  $s = a \otimes a_V$ , an object  $a$ , and an object  $r = 1$ . The output is the coclosed coevaluation morphism  $\text{coclcov}_a : a \otimes a_V \rightarrow 1$ .

### 1.10.9 CoTraceMap (for IsCapCategoryMorphism)

▷ CoTraceMap( $alpha$ ) (attribute)

**Returns:** a morphism in  $\text{Hom}(1, 1)$ .

The argument is an endomorphism  $\alpha : a \rightarrow a$ . The output is the cotrace morphism  $\text{cotrace}_\alpha : 1 \rightarrow 1$ .

### 1.10.10 CoRankMorphism (for IsCapCategoryObject)

▷ CoRankMorphism( $a$ ) (attribute)

**Returns:** a morphism in  $\text{Hom}(1, 1)$ .

The argument is an object  $a$ . The output is the corank morphism  $\text{corank}_a : 1 \rightarrow 1$ .

### 1.10.11 MorphismToCoBidual (for IsCapCategoryObject)

▷ MorphismToCoBidual( $a$ ) (attribute)

**Returns:** a morphism in  $\text{Hom}(a, (a_V)_V)$ .

The argument is an object  $a$ . The output is the inverse of the morphism from the cobidual  $a \rightarrow (a_V)_V$ .

### 1.10.12 MorphismToCoBidualWithGivenCoBidual (for IsCapCategoryObject, IsCapCategoryObject)

▷ `MorphismToCoBidualWithGivenCoBidual(a, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(a, (a_V)_V)$ .

The argument is an object  $a$ , and an object  $r = (a_V)_V$ . The output is the inverse of the morphism from the cobidual  $a \rightarrow (a_V)_V$ .

## 1.11 Convenience Methods

### 1.11.1 InternalHom (for IsCapCategoryCell, IsCapCategoryCell)

▷ `InternalHom(a, b)` (operation)

**Returns:** a cell

This is a convenience method. The arguments are two cells  $a, b$ . The output is the internal hom cell. If  $a, b$  are two CAP objects the output is the internal Hom object  $\underline{\text{Hom}}(a, b)$ . If at least one of the arguments is a CAP morphism the output is a CAP morphism, namely the internal hom on morphisms, where any object is replaced by its identity morphism.

### 1.11.2 InternalCoHom (for IsCapCategoryCell, IsCapCategoryCell)

▷ `InternalCoHom(a, b)` (operation)

**Returns:** a cell

This is a convenience method. The arguments are two cells  $a, b$ . The output is the internal cohom cell. If  $a, b$  are two CAP objects the output is the internal cohom object  $\underline{\text{coHom}}(a, b)$ . If at least one of the arguments is a CAP morphism the output is a CAP morphism, namely the internal cohom on morphisms, where any object is replaced by its identity morphism.

## 1.12 Add-methods

### 1.12.1 AddLeftDistributivityExpanding (for IsCapCategory, IsFunction)

▷ `AddLeftDistributivityExpanding(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `LeftDistributivityExpanding`.  $F : (a, L) \mapsto \text{LeftDistributivityExpanding}(a, L)$ .

### 1.12.2 AddLeftDistributivityExpandingWithGivenObjects (for IsCapCategory, IsFunction)

▷ `AddLeftDistributivityExpandingWithGivenObjects(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `LeftDistributivityExpandingWithGivenObjects`.  $F : (s, a, L, r) \mapsto \text{LeftDistributivityExpandingWithGivenObjects}(s, a, L, r)$ .

### 1.12.3 AddLeftDistributivityFactoring (for IsCapCategory, IsFunction)

▷ AddLeftDistributivityFactoring( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftDistributivityFactoring.  $F : (a, L) \mapsto \text{LeftDistributivityFactoring}(a, L)$ .

### 1.12.4 AddLeftDistributivityFactoringWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddLeftDistributivityFactoringWithGivenObjects( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftDistributivityFactoringWithGivenObjects.  $F : (s, a, L, r) \mapsto \text{LeftDistributivityFactoringWithGivenObjects}(s, a, L, r)$ .

### 1.12.5 AddRightDistributivityExpanding (for IsCapCategory, IsFunction)

▷ AddRightDistributivityExpanding( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation RightDistributivityExpanding.  $F : (L, a) \mapsto \text{RightDistributivityExpanding}(L, a)$ .

### 1.12.6 AddRightDistributivityExpandingWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddRightDistributivityExpandingWithGivenObjects( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation RightDistributivityExpandingWithGivenObjects.  $F : (s, L, a, r) \mapsto \text{RightDistributivityExpandingWithGivenObjects}(s, L, a, r)$ .

### 1.12.7 AddRightDistributivityFactoring (for IsCapCategory, IsFunction)

▷ AddRightDistributivityFactoring( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation RightDistributivityFactoring.  $F : (L, a) \mapsto \text{RightDistributivityFactoring}(L, a)$ .

### 1.12.8 AddRightDistributivityFactoringWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddRightDistributivityFactoringWithGivenObjects( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `RightDistributivityFactoringWithGivenObjects`.  $F : (s, L, a, r) \mapsto \text{RightDistributivityFactoringWithGivenObjects}(s, L, a, r)$ .

### 1.12.9 AddBraiding (for IsCapCategory, IsFunction)

▷ `AddBraiding(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `Braiding`.  $F : (a, b) \mapsto \text{Braiding}(a, b)$ .

### 1.12.10 AddBraidingInverse (for IsCapCategory, IsFunction)

▷ `AddBraidingInverse(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `BraidingInverse`.  $F : (a, b) \mapsto \text{BraidingInverse}(a, b)$ .

### 1.12.11 AddBraidingInverseWithGivenTensorProducts (for IsCapCategory, IsFunction)

▷ `AddBraidingInverseWithGivenTensorProducts(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `BraidingInverseWithGivenTensorProducts`.  $F : (s, a, b, r) \mapsto \text{BraidingInverseWithGivenTensorProducts}(s, a, b, r)$ .

### 1.12.12 AddBraidingWithGivenTensorProducts (for IsCapCategory, IsFunction)

▷ `AddBraidingWithGivenTensorProducts(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `BraidingWithGivenTensorProducts`.  $F : (s, a, b, r) \mapsto \text{BraidingWithGivenTensorProducts}(s, a, b, r)$ .

### 1.12.13 AddCoevaluationMorphism (for IsCapCategory, IsFunction)

▷ `AddCoevaluationMorphism(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `CoevaluationMorphism`.  $F : (a, b) \mapsto \text{CoevaluationMorphism}(a, b)$ .

### 1.12.14 AddCoevaluationMorphismWithGivenRange (for IsCapCategory, IsFunction)

▷ `AddCoevaluationMorphismWithGivenRange(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `CoevaluationMorphismWithGivenRange`.  $F : (a, b, r) \mapsto \text{CoevaluationMorphismWithGivenRange}(a, b, r)$ .

### 1.12.15 AddDualOnMorphisms (for IsCapCategory, IsFunction)

▷ `AddDualOnMorphisms(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `DualOnMorphisms`.  $F : (alpha) \mapsto \text{DualOnMorphisms}(alpha)$ .

### 1.12.16 AddDualOnMorphismsWithGivenDuals (for IsCapCategory, IsFunction)

▷ `AddDualOnMorphismsWithGivenDuals(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `DualOnMorphismsWithGivenDuals`.  $F : (s, alpha, r) \mapsto \text{DualOnMorphismsWithGivenDuals}(s, alpha, r)$ .

### 1.12.17 AddDualOnObjects (for IsCapCategory, IsFunction)

▷ `AddDualOnObjects(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `DualOnObjects`.  $F : (a) \mapsto \text{DualOnObjects}(a)$ .

### 1.12.18 AddEvaluationForDual (for IsCapCategory, IsFunction)

▷ `AddEvaluationForDual(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `EvaluationForDual`.  $F : (a) \mapsto \text{EvaluationForDual}(a)$ .

### 1.12.19 AddEvaluationForDualWithGivenTensorProduct (for IsCapCategory, IsFunction)

▷ `AddEvaluationForDualWithGivenTensorProduct(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `EvaluationForDualWithGivenTensorProduct`.  $F : (s, a, r) \mapsto \text{EvaluationForDualWithGivenTensorProduct}(s, a, r)$ .

### 1.12.20 AddEvaluationMorphism (for IsCapCategory, IsFunction)

▷ `AddEvaluationMorphism(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `EvaluationMorphism`.  $F : (a, b) \mapsto \text{EvaluationMorphism}(a, b)$ .

### 1.12.21 AddEvaluationMorphismWithGivenSource (for IsCapCategory, IsFunction)

▷ AddEvaluationMorphismWithGivenSource( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation EvaluationMorphismWithGivenSource.  $F : (a, b, s) \mapsto \text{EvaluationMorphismWithGivenSource}(a, b, s)$ .

### 1.12.22 AddInternalHomOnMorphisms (for IsCapCategory, IsFunction)

▷ AddInternalHomOnMorphisms( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation InternalHomOnMorphisms.  $F : (alpha, beta) \mapsto \text{InternalHomOnMorphisms}(alpha, beta)$ .

### 1.12.23 AddInternalHomOnMorphismsWithGivenInternalHoms (for IsCapCategory, IsFunction)

▷ AddInternalHomOnMorphismsWithGivenInternalHoms( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation InternalHomOnMorphismsWithGivenInternalHoms.  $F : (s, alpha, beta, r) \mapsto \text{InternalHomOnMorphismsWithGivenInternalHoms}(s, alpha, beta, r)$ .

### 1.12.24 AddInternalHomOnObjects (for IsCapCategory, IsFunction)

▷ AddInternalHomOnObjects( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation InternalHomOnObjects.  $F : (a, b) \mapsto \text{InternalHomOnObjects}(a, b)$ .

### 1.12.25 AddInternalHomToTensorProductAdjunctionMap (for IsCapCategory, IsFunction)

▷ AddInternalHomToTensorProductAdjunctionMap( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation InternalHomToTensorProductAdjunctionMap.  $F : (b, c, g) \mapsto \text{InternalHomToTensorProductAdjunctionMap}(b, c, g)$ .

### 1.12.26 AddInternalHomToTensorProductAdjunctionMapWithGivenTensorProduct (for IsCapCategory, IsFunction)

▷ AddInternalHomToTensorProductAdjunctionMapWithGivenTensorProduct( $C, F$ ) (operation)

**Returns:** nothing



The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `InternalHomToTensorProductAdjunctionMapWithGivenTensorProduct`.  $F : (b, c, g, t) \mapsto \text{InternalHomToTensorProductAdjunctionMapWithGivenTensorProduct}(b, c, g, t)$ .

### 1.12.27 AddIsomorphismFromDualObjectToInternalHomIntoTensorUnit (for IsCapCategory, IsFunction)

▷ `AddIsomorphismFromDualObjectToInternalHomIntoTensorUnit(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `IsomorphismFromDualObjectToInternalHomIntoTensorUnit`.  $F : (a) \mapsto \text{IsomorphismFromDualObjectToInternalHomIntoTensorUnit}(a)$ .

### 1.12.28 AddIsomorphismFromInternalHomIntoTensorUnitToDualObject (for IsCapCategory, IsFunction)

▷ `AddIsomorphismFromInternalHomIntoTensorUnitToDualObject(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `IsomorphismFromInternalHomIntoTensorUnitToDualObject`.  $F : (a) \mapsto \text{IsomorphismFromInternalHomIntoTensorUnitToDualObject}(a)$ .

### 1.12.29 AddIsomorphismFromInternalHomToObject (for IsCapCategory, IsFunction)

▷ `AddIsomorphismFromInternalHomToObject(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `IsomorphismFromInternalHomToObject`.  $F : (a) \mapsto \text{IsomorphismFromInternalHomToObject}(a)$ .

### 1.12.30 AddIsomorphismFromInternalHomToObjectWithGivenInternalHom (for IsCapCategory, IsFunction)

▷ `AddIsomorphismFromInternalHomToObjectWithGivenInternalHom(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `IsomorphismFromInternalHomToObjectWithGivenInternalHom`.  $F : (a, s) \mapsto \text{IsomorphismFromInternalHomToObjectWithGivenInternalHom}(a, s)$ .

### 1.12.31 AddIsomorphismFromObjectToInternalHom (for IsCapCategory, IsFunction)

▷ `AddIsomorphismFromObjectToInternalHom(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `IsomorphismFromObjectToInternalHom`.  $F : (a) \mapsto \text{IsomorphismFromObjectToInternalHom}(a)$ .

### 1.12.32 AddIsomorphismFromObjectToInternalHomWithGivenInternalHom (for IsCapCategory, IsFunction)

▷ `AddIsomorphismFromObjectToInternalHomWithGivenInternalHom(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `IsomorphismFromObjectToInternalHomWithGivenInternalHom`.  $F : (a, r) \mapsto \text{IsomorphismFromObjectToInternalHomWithGivenInternalHom}(a, r)$ .

### 1.12.33 AddLambdaElimination (for IsCapCategory, IsFunction)

▷ `AddLambdaElimination(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `LambdaElimination`.  $F : (a, b, \alpha) \mapsto \text{LambdaElimination}(a, b, \alpha)$ .

### 1.12.34 AddLambdaIntroduction (for IsCapCategory, IsFunction)

▷ `AddLambdaIntroduction(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `LambdaIntroduction`.  $F : (\alpha) \mapsto \text{LambdaIntroduction}(\alpha)$ .

### 1.12.35 AddMonoidalPostComposeMorphism (for IsCapCategory, IsFunction)

▷ `AddMonoidalPostComposeMorphism(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MonoidalPostComposeMorphism`.  $F : (a, b, c) \mapsto \text{MonoidalPostComposeMorphism}(a, b, c)$ .

### 1.12.36 AddMonoidalPostComposeMorphismWithGivenObjects (for IsCapCategory, IsFunction)

▷ `AddMonoidalPostComposeMorphismWithGivenObjects(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MonoidalPostComposeMorphismWithGivenObjects`.  $F : (s, a, b, c, r) \mapsto \text{MonoidalPostComposeMorphismWithGivenObjects}(s, a, b, c, r)$ .

### 1.12.37 AddMonoidalPreComposeMorphism (for IsCapCategory, IsFunction)

▷ AddMonoidalPreComposeMorphism( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation MonoidalPreComposeMorphism.  $F : (a, b, c) \mapsto \text{MonoidalPreComposeMorphism}(a, b, c)$ .

### 1.12.38 AddMonoidalPreComposeMorphismWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddMonoidalPreComposeMorphismWithGivenObjects( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation MonoidalPreComposeMorphismWithGivenObjects.  $F : (s, a, b, c, r) \mapsto \text{MonoidalPreComposeMorphismWithGivenObjects}(s, a, b, c, r)$ .

### 1.12.39 AddMorphismFromTensorProductToInternalHom (for IsCapCategory, IsFunction)

▷ AddMorphismFromTensorProductToInternalHom( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation MorphismFromTensorProductToInternalHom.  $F : (a, b) \mapsto \text{MorphismFromTensorProductToInternalHom}(a, b)$ .

### 1.12.40 AddMorphismFromTensorProductToInternalHomWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddMorphismFromTensorProductToInternalHomWithGivenObjects( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation MorphismFromTensorProductToInternalHomWithGivenObjects.  $F : (s, a, b, r) \mapsto \text{MorphismFromTensorProductToInternalHomWithGivenObjects}(s, a, b, r)$ .

### 1.12.41 AddMorphismToBidual (for IsCapCategory, IsFunction)

▷ AddMorphismToBidual( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation MorphismToBidual.  $F : (a) \mapsto \text{MorphismToBidual}(a)$ .

### 1.12.42 AddMorphismToBidualWithGivenBidual (for IsCapCategory, IsFunction)

▷ AddMorphismToBidualWithGivenBidual( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MorphismToBidualWithGivenBidual`.  $F : (a, r) \mapsto \text{MorphismToBidualWithGivenBidual}(a, r)$ .

### 1.12.43 AddTensorProductDualityCompatibilityMorphism (for IsCapCategory, IsFunction)

▷ `AddTensorProductDualityCompatibilityMorphism(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `TensorProductDualityCompatibilityMorphism`.  $F : (a, b) \mapsto \text{TensorProductDualityCompatibilityMorphism}(a, b)$ .

### 1.12.44 AddTensorProductDualityCompatibilityMorphismWithGivenObjects (for IsCapCategory, IsFunction)

▷ `AddTensorProductDualityCompatibilityMorphismWithGivenObjects(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `TensorProductDualityCompatibilityMorphismWithGivenObjects`.  $F : (s, a, b, r) \mapsto \text{TensorProductDualityCompatibilityMorphismWithGivenObjects}(s, a, b, r)$ .

### 1.12.45 AddTensorProductInternalHomCompatibilityMorphism (for IsCapCategory, IsFunction)

▷ `AddTensorProductInternalHomCompatibilityMorphism(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `TensorProductInternalHomCompatibilityMorphism`.  $F : (list) \mapsto \text{TensorProductInternalHomCompatibilityMorphism}(list)$ .

### 1.12.46 AddTensorProductInternalHomCompatibilityMorphismWithGivenObjects (for IsCapCategory, IsFunction)

▷ `AddTensorProductInternalHomCompatibilityMorphismWithGivenObjects(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `TensorProductInternalHomCompatibilityMorphismWithGivenObjects`.  $F : (source, list, range) \mapsto \text{TensorProductInternalHomCompatibilityMorphismWithGivenObjects}(source, list, range)$ .

### 1.12.47 AddTensorProductToInternalHomAdjunctionMap (for IsCapCategory, IsFunction)

▷ AddTensorProductToInternalHomAdjunctionMap( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation TensorProductToInternalHomAdjunctionMap.  $F : (a, b, f) \mapsto \text{TensorProductToInternalHomAdjunctionMap}(a, b, f)$ .

### 1.12.48 AddTensorProductToInternalHomAdjunctionMapWithGivenInternalHom (for IsCapCategory, IsFunction)

▷ AddTensorProductToInternalHomAdjunctionMapWithGivenInternalHom( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation TensorProductToInternalHomAdjunctionMapWithGivenInternalHom.  $F : (a, b, f, i) \mapsto \text{TensorProductToInternalHomAdjunctionMapWithGivenInternalHom}(a, b, f, i)$ .

### 1.12.49 AddUniversalPropertyOfDual (for IsCapCategory, IsFunction)

▷ AddUniversalPropertyOfDual( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation UniversalPropertyOfDual.  $F : (t, a, alpha) \mapsto \text{UniversalPropertyOfDual}(t, a, alpha)$ .

### 1.12.50 AddCoDualOnMorphisms (for IsCapCategory, IsFunction)

▷ AddCoDualOnMorphisms( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation CoDualOnMorphisms.  $F : (alpha) \mapsto \text{CoDualOnMorphisms}(alpha)$ .

### 1.12.51 AddCoDualOnMorphismsWithGivenCoDuals (for IsCapCategory, IsFunction)

▷ AddCoDualOnMorphismsWithGivenCoDuals( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation CoDualOnMorphismsWithGivenCoDuals.  $F : (s, alpha, r) \mapsto \text{CoDualOnMorphismsWithGivenCoDuals}(s, alpha, r)$ .

### 1.12.52 AddCoDualOnObjects (for IsCapCategory, IsFunction)

▷ AddCoDualOnObjects( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `CoDualOnObjects`.  $F : (a) \mapsto \text{CoDualOnObjects}(a)$ .

### 1.12.53 AddCoDualityTensorProductCompatibilityMorphism (for IsCapCategory, IsFunction)

▷ `AddCoDualityTensorProductCompatibilityMorphism(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `CoDualityTensorProductCompatibilityMorphism`.  $F : (a, b) \mapsto \text{CoDualityTensorProductCompatibilityMorphism}(a, b)$ .

### 1.12.54 AddCoDualityTensorProductCompatibilityMorphismWithGivenObjects (for IsCapCategory, IsFunction)

▷ `AddCoDualityTensorProductCompatibilityMorphismWithGivenObjects(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `CoDualityTensorProductCompatibilityMorphismWithGivenObjects`.  $F : (s, a, b, r) \mapsto \text{CoDualityTensorProductCompatibilityMorphismWithGivenObjects}(s, a, b, r)$ .

### 1.12.55 AddCoLambdaElimination (for IsCapCategory, IsFunction)

▷ `AddCoLambdaElimination(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `CoLambdaElimination`.  $F : (a, b, alpha) \mapsto \text{CoLambdaElimination}(a, b, alpha)$ .

### 1.12.56 AddCoLambdaIntroduction (for IsCapCategory, IsFunction)

▷ `AddCoLambdaIntroduction(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `CoLambdaIntroduction`.  $F : (alpha) \mapsto \text{CoLambdaIntroduction}(alpha)$ .

### 1.12.57 AddCoclosedCoevaluationMorphism (for IsCapCategory, IsFunction)

▷ `AddCoclosedCoevaluationMorphism(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `CoclosedCoevaluationMorphism`.  $F : (a, b) \mapsto \text{CoclosedCoevaluationMorphism}(a, b)$ .

### 1.12.58 AddCoclosedCoevaluationMorphismWithGivenSource (for IsCapCategory, IsFunction)

▷ AddCoclosedCoevaluationMorphismWithGivenSource( $\mathcal{C}$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $\mathcal{C}$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation CoclosedCoevaluationMorphismWithGivenSource.  $F : (a, b, s) \mapsto \text{CoclosedCoevaluationMorphismWithGivenSource}(a, b, s)$ .

### 1.12.59 AddCoclosedEvaluationForCoDual (for IsCapCategory, IsFunction)

▷ AddCoclosedEvaluationForCoDual( $\mathcal{C}$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $\mathcal{C}$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation CoclosedEvaluationForCoDual.  $F : (a) \mapsto \text{CoclosedEvaluationForCoDual}(a)$ .

### 1.12.60 AddCoclosedEvaluationForCoDualWithGivenTensorProduct (for IsCapCategory, IsFunction)

▷ AddCoclosedEvaluationForCoDualWithGivenTensorProduct( $\mathcal{C}$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $\mathcal{C}$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation CoclosedEvaluationForCoDualWithGivenTensorProduct.  $F : (s, a, r) \mapsto \text{CoclosedEvaluationForCoDualWithGivenTensorProduct}(s, a, r)$ .

### 1.12.61 AddCoclosedEvaluationMorphism (for IsCapCategory, IsFunction)

▷ AddCoclosedEvaluationMorphism( $\mathcal{C}$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $\mathcal{C}$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation CoclosedEvaluationMorphism.  $F : (a, b) \mapsto \text{CoclosedEvaluationMorphism}(a, b)$ .

### 1.12.62 AddCoclosedEvaluationMorphismWithGivenRange (for IsCapCategory, IsFunction)

▷ AddCoclosedEvaluationMorphismWithGivenRange( $\mathcal{C}$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $\mathcal{C}$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation CoclosedEvaluationMorphismWithGivenRange.  $F : (a, b, r) \mapsto \text{CoclosedEvaluationMorphismWithGivenRange}(a, b, r)$ .

### 1.12.63 AddInternalCoHomOnMorphisms (for IsCapCategory, IsFunction)

▷ AddInternalCoHomOnMorphisms( $\mathcal{C}$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `InternalCoHomOnMorphisms`.  $F : (\alpha, \beta) \mapsto \text{InternalCoHomOnMorphisms}(\alpha, \beta)$ .

### 1.12.64 AddInternalCoHomOnMorphismsWithGivenInternalCoHoms (for IsCapCategory, IsFunction)

▷ `AddInternalCoHomOnMorphismsWithGivenInternalCoHoms(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `InternalCoHomOnMorphismsWithGivenInternalCoHoms`.  $F : (s, \alpha, \beta, r) \mapsto \text{InternalCoHomOnMorphismsWithGivenInternalCoHoms}(s, \alpha, \beta, r)$ .

### 1.12.65 AddInternalCoHomOnObjects (for IsCapCategory, IsFunction)

▷ `AddInternalCoHomOnObjects(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `InternalCoHomOnObjects`.  $F : (a, b) \mapsto \text{InternalCoHomOnObjects}(a, b)$ .

### 1.12.66 AddInternalCoHomTensorProductCompatibilityMorphism (for IsCapCategory, IsFunction)

▷ `AddInternalCoHomTensorProductCompatibilityMorphism(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `InternalCoHomTensorProductCompatibilityMorphism`.  $F : (list) \mapsto \text{InternalCoHomTensorProductCompatibilityMorphism}(list)$ .

### 1.12.67 AddInternalCoHomTensorProductCompatibilityMorphismWithGivenObjects (for IsCapCategory, IsFunction)

▷ `AddInternalCoHomTensorProductCompatibilityMorphismWithGivenObjects(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `InternalCoHomTensorProductCompatibilityMorphismWithGivenObjects`.  $F : (source, list, range) \mapsto \text{InternalCoHomTensorProductCompatibilityMorphismWithGivenObjects}(source, list, range)$ .

### 1.12.68 AddInternalCoHomToTensorProductAdjunctionMap (for IsCapCategory, IsFunction)

▷ `AddInternalCoHomToTensorProductAdjunctionMap(C, F)` (operation)

**Returns:** nothing



The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `InternalCoHomToTensorProductAdjunctionMap`.  $F : (a, b, f) \mapsto \text{InternalCoHomToTensorProductAdjunctionMap}(a, b, f)$ .

### 1.12.69 AddInternalCoHomToTensorProductAdjunctionMapWithGivenTensorProduct (for IsCapCategory, IsFunction)

▷ `AddInternalCoHomToTensorProductAdjunctionMapWithGivenTensorProduct(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `InternalCoHomToTensorProductAdjunctionMapWithGivenTensorProduct`.  $F : (a, b, f, t) \mapsto \text{InternalCoHomToTensorProductAdjunctionMapWithGivenTensorProduct}(a, b, f, t)$ .

### 1.12.70 AddIsomorphismFromCoDualObjectToInternalCoHomFromTensorUnit (for IsCapCategory, IsFunction)

▷ `AddIsomorphismFromCoDualObjectToInternalCoHomFromTensorUnit(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `IsomorphismFromCoDualObjectToInternalCoHomFromTensorUnit`.  $F : (a) \mapsto \text{IsomorphismFromCoDualObjectToInternalCoHomFromTensorUnit}(a)$ .

### 1.12.71 AddIsomorphismFromInternalCoHomFromTensorUnitToCoDualObject (for IsCapCategory, IsFunction)

▷ `AddIsomorphismFromInternalCoHomFromTensorUnitToCoDualObject(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `IsomorphismFromInternalCoHomFromTensorUnitToCoDualObject`.  $F : (a) \mapsto \text{IsomorphismFromInternalCoHomFromTensorUnitToCoDualObject}(a)$ .

### 1.12.72 AddIsomorphismFromInternalCoHomToObject (for IsCapCategory, IsFunction)

▷ `AddIsomorphismFromInternalCoHomToObject(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `IsomorphismFromInternalCoHomToObject`.  $F : (a) \mapsto \text{IsomorphismFromInternalCoHomToObject}(a)$ .

### 1.12.73 AddIsomorphismFromInternalCoHomToObjectWithGivenInternalCoHom (for IsCapCategory, IsFunction)

▷ AddIsomorphismFromInternalCoHomToObjectWithGivenInternalCoHom( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation IsomorphismFromInternalCoHomToObjectWithGivenInternalCoHom.  $F : (a, s) \mapsto \text{IsomorphismFromInternalCoHomToObjectWithGivenInternalCoHom}(a, s)$ .

### 1.12.74 AddIsomorphismFromObjectToInternalCoHom (for IsCapCategory, IsFunction)

▷ AddIsomorphismFromObjectToInternalCoHom( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation IsomorphismFromObjectToInternalCoHom.  $F : (a) \mapsto \text{IsomorphismFromObjectToInternalCoHom}(a)$ .

### 1.12.75 AddIsomorphismFromObjectToInternalCoHomWithGivenInternalCoHom (for IsCapCategory, IsFunction)

▷ AddIsomorphismFromObjectToInternalCoHomWithGivenInternalCoHom( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation IsomorphismFromObjectToInternalCoHomWithGivenInternalCoHom.  $F : (a, r) \mapsto \text{IsomorphismFromObjectToInternalCoHomWithGivenInternalCoHom}(a, r)$ .

### 1.12.76 AddMonoidalPostCoComposeMorphism (for IsCapCategory, IsFunction)

▷ AddMonoidalPostCoComposeMorphism( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation MonoidalPostCoComposeMorphism.  $F : (a, b, c) \mapsto \text{MonoidalPostCoComposeMorphism}(a, b, c)$ .

### 1.12.77 AddMonoidalPostCoComposeMorphismWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddMonoidalPostCoComposeMorphismWithGivenObjects( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation MonoidalPostCoComposeMorphismWithGivenObjects.  $F : (s, a, b, c, r) \mapsto \text{MonoidalPostCoComposeMorphismWithGivenObjects}(s, a, b, c, r)$ .

### 1.12.78 AddMonoidalPreCoComposeMorphism (for IsCapCategory, IsFunction)

▷ AddMonoidalPreCoComposeMorphism( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation MonoidalPreCoComposeMorphism.  $F : (a, b, c) \mapsto \text{MonoidalPreCoComposeMorphism}(a, b, c)$ .

### 1.12.79 AddMonoidalPreCoComposeMorphismWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddMonoidalPreCoComposeMorphismWithGivenObjects( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation MonoidalPreCoComposeMorphismWithGivenObjects.  $F : (s, a, b, c, r) \mapsto \text{MonoidalPreCoComposeMorphismWithGivenObjects}(s, a, b, c, r)$ .

### 1.12.80 AddMorphismFromCoBidual (for IsCapCategory, IsFunction)

▷ AddMorphismFromCoBidual( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation MorphismFromCoBidual.  $F : (a) \mapsto \text{MorphismFromCoBidual}(a)$ .

### 1.12.81 AddMorphismFromCoBidualWithGivenCoBidual (for IsCapCategory, IsFunction)

▷ AddMorphismFromCoBidualWithGivenCoBidual( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation MorphismFromCoBidualWithGivenCoBidual.  $F : (a, s) \mapsto \text{MorphismFromCoBidualWithGivenCoBidual}(a, s)$ .

### 1.12.82 AddMorphismFromInternalCoHomToTensorProduct (for IsCapCategory, IsFunction)

▷ AddMorphismFromInternalCoHomToTensorProduct( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation MorphismFromInternalCoHomToTensorProduct.  $F : (a, b) \mapsto \text{MorphismFromInternalCoHomToTensorProduct}(a, b)$ .

### 1.12.83 AddMorphismFromInternalCoHomToTensorProductWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddMorphismFromInternalCoHomToTensorProductWithGivenObjects( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MorphismFromInternalCoHomToTensorProductWithGivenObjects`.  $F : (s, a, b, r) \mapsto \text{MorphismFromInternalCoHomToTensorProductWithGivenObjects}(s, a, b, r)$ .

#### 1.12.84 AddTensorProductToInternalCoHomAdjunctionMap (for IsCapCategory, IsFunction)

▷ `AddTensorProductToInternalCoHomAdjunctionMap(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `TensorProductToInternalCoHomAdjunctionMap`.  $F : (c, b, g) \mapsto \text{TensorProductToInternalCoHomAdjunctionMap}(c, b, g)$ .

#### 1.12.85 AddTensorProductToInternalCoHomAdjunctionMapWithGivenInternalCoHom (for IsCapCategory, IsFunction)

▷ `AddTensorProductToInternalCoHomAdjunctionMapWithGivenInternalCoHom(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `TensorProductToInternalCoHomAdjunctionMapWithGivenInternalCoHom`.  $F : (c, b, g, i) \mapsto \text{TensorProductToInternalCoHomAdjunctionMapWithGivenInternalCoHom}(c, b, g, i)$ .

#### 1.12.86 AddUniversalPropertyOfCoDual (for IsCapCategory, IsFunction)

▷ `AddUniversalPropertyOfCoDual(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `UniversalPropertyOfCoDual`.  $F : (t, a, alpha) \mapsto \text{UniversalPropertyOfCoDual}(t, a, alpha)$ .

#### 1.12.87 AddAssociatorLeftToRight (for IsCapCategory, IsFunction)

▷ `AddAssociatorLeftToRight(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `AssociatorLeftToRight`.  $F : (a, b, c) \mapsto \text{AssociatorLeftToRight}(a, b, c)$ .

#### 1.12.88 AddAssociatorLeftToRightWithGivenTensorProducts (for IsCapCategory, IsFunction)

▷ `AddAssociatorLeftToRightWithGivenTensorProducts(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `AssociatorLeftToRightWithGivenTensorProducts`.  $F : (s, a, b, c, r) \mapsto \text{AssociatorLeftToRightWithGivenTensorProducts}(s, a, b, c, r)$ .

### 1.12.89 AddAssociatorRightToLeft (for IsCapCategory, IsFunction)

▷ `AddAssociatorRightToLeft(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `AssociatorRightToLeft`.  $F : (a, b, c) \mapsto \text{AssociatorRightToLeft}(a, b, c)$ .

### 1.12.90 AddAssociatorRightToLeftWithGivenTensorProducts (for IsCapCategory, IsFunction)

▷ `AddAssociatorRightToLeftWithGivenTensorProducts(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `AssociatorRightToLeftWithGivenTensorProducts`.  $F : (s, a, b, c, r) \mapsto \text{AssociatorRightToLeftWithGivenTensorProducts}(s, a, b, c, r)$ .

### 1.12.91 AddLeftUnitor (for IsCapCategory, IsFunction)

▷ `AddLeftUnitor(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `LeftUnitor`.  $F : (a) \mapsto \text{LeftUnitor}(a)$ .

### 1.12.92 AddLeftUnitorInverse (for IsCapCategory, IsFunction)

▷ `AddLeftUnitorInverse(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `LeftUnitorInverse`.  $F : (a) \mapsto \text{LeftUnitorInverse}(a)$ .

### 1.12.93 AddLeftUnitorInverseWithGivenTensorProduct (for IsCapCategory, IsFunction)

▷ `AddLeftUnitorInverseWithGivenTensorProduct(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `LeftUnitorInverseWithGivenTensorProduct`.  $F : (a, r) \mapsto \text{LeftUnitorInverseWithGivenTensorProduct}(a, r)$ .

### 1.12.94 AddLeftUnitorWithGivenTensorProduct (for IsCapCategory, IsFunction)

▷ `AddLeftUnitorWithGivenTensorProduct(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `LeftUnitorWithGivenTensorProduct`.  $F : (a,s) \mapsto \text{LeftUnitorWithGivenTensorProduct}(a,s)$ .

### 1.12.95 AddRightUnitor (for IsCapCategory, IsFunction)

▷ `AddRightUnitor(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `RightUnitor`.  $F : (a) \mapsto \text{RightUnitor}(a)$ .

### 1.12.96 AddRightUnitorInverse (for IsCapCategory, IsFunction)

▷ `AddRightUnitorInverse(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `RightUnitorInverse`.  $F : (a) \mapsto \text{RightUnitorInverse}(a)$ .

### 1.12.97 AddRightUnitorInverseWithGivenTensorProduct (for IsCapCategory, IsFunction)

▷ `AddRightUnitorInverseWithGivenTensorProduct(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `RightUnitorInverseWithGivenTensorProduct`.  $F : (a,r) \mapsto \text{RightUnitorInverseWithGivenTensorProduct}(a,r)$ .

### 1.12.98 AddRightUnitorWithGivenTensorProduct (for IsCapCategory, IsFunction)

▷ `AddRightUnitorWithGivenTensorProduct(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `RightUnitorWithGivenTensorProduct`.  $F : (a,s) \mapsto \text{RightUnitorWithGivenTensorProduct}(a,s)$ .

### 1.12.99 AddTensorProductOnMorphisms (for IsCapCategory, IsFunction)

▷ `AddTensorProductOnMorphisms(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `TensorProductOnMorphisms`.  $F : (alpha,beta) \mapsto \text{TensorProductOnMorphisms}(alpha,beta)$ .

### 1.12.100 AddTensorProductOnMorphismsWithGivenTensorProducts (for IsCapCategory, IsFunction)

▷ `AddTensorProductOnMorphismsWithGivenTensorProducts(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `TensorProductOnMorphismsWithGivenTensorProducts`.  $F : (s, \alpha, \beta, r) \mapsto \text{TensorProductOnMorphismsWithGivenTensorProducts}(s, \alpha, \beta, r)$ .

### 1.12.101 AddCoevaluationForDual (for IsCapCategory, IsFunction)

▷ `AddCoevaluationForDual(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `CoevaluationForDual`.  $F : (a) \mapsto \text{CoevaluationForDual}(a)$ .

### 1.12.102 AddCoevaluationForDualWithGivenTensorProduct (for IsCapCategory, IsFunction)

▷ `AddCoevaluationForDualWithGivenTensorProduct(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `CoevaluationForDualWithGivenTensorProduct`.  $F : (s, a, r) \mapsto \text{CoevaluationForDualWithGivenTensorProduct}(s, a, r)$ .

### 1.12.103 AddIsomorphismFromInternalHomToTensorProductWithDualObject (for IsCapCategory, IsFunction)

▷ `AddIsomorphismFromInternalHomToTensorProductWithDualObject(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `IsomorphismFromInternalHomToTensorProductWithDualObject`.  $F : (a, b) \mapsto \text{IsomorphismFromInternalHomToTensorProductWithDualObject}(a, b)$ .

### 1.12.104 AddIsomorphismFromTensorProductWithDualObjectToInternalHom (for IsCapCategory, IsFunction)

▷ `AddIsomorphismFromTensorProductWithDualObjectToInternalHom(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `IsomorphismFromTensorProductWithDualObjectToInternalHom`.  $F : (a, b) \mapsto \text{IsomorphismFromTensorProductWithDualObjectToInternalHom}(a, b)$ .

### 1.12.105 AddMorphismFromBidual (for IsCapCategory, IsFunction)

▷ `AddMorphismFromBidual(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MorphismFromBidual`.  $F : (a) \mapsto \text{MorphismFromBidual}(a)$ .

### 1.12.106 AddMorphismFromBidualWithGivenBidual (for IsCapCategory, IsFunction)

▷ AddMorphismFromBidualWithGivenBidual( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation MorphismFromBidualWithGivenBidual.  $F : (a, s) \mapsto \text{MorphismFromBidualWithGivenBidual}(a, s)$ .

### 1.12.107 AddMorphismFromInternalHomToTensorProduct (for IsCapCategory, IsFunction)

▷ AddMorphismFromInternalHomToTensorProduct( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation MorphismFromInternalHomToTensorProduct.  $F : (a, b) \mapsto \text{MorphismFromInternalHomToTensorProduct}(a, b)$ .

### 1.12.108 AddMorphismFromInternalHomToTensorProductWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddMorphismFromInternalHomToTensorProductWithGivenObjects( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation MorphismFromInternalHomToTensorProductWithGivenObjects.  $F : (s, a, b, r) \mapsto \text{MorphismFromInternalHomToTensorProductWithGivenObjects}(s, a, b, r)$ .

### 1.12.109 AddRankMorphism (for IsCapCategory, IsFunction)

▷ AddRankMorphism( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation RankMorphism.  $F : (a) \mapsto \text{RankMorphism}(a)$ .

### 1.12.110 AddTensorProductInternalHomCompatibilityMorphismInverse (for IsCapCategory, IsFunction)

▷ AddTensorProductInternalHomCompatibilityMorphismInverse( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation TensorProductInternalHomCompatibilityMorphismInverse.  $F : (list) \mapsto \text{TensorProductInternalHomCompatibilityMorphismInverse}(list)$ .



### 1.12.111 AddTensorProductInternalHomCompatibilityMorphismInverseWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddTensorProductInternalHomCompatibilityMorphismInverseWithGivenObjects( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation TensorProductInternalHomCompatibilityMorphismInverseWithGivenObjects.  $F : (source, list, range) \mapsto \text{TensorProductInternalHomCompatibilityMorphismInverseWithGivenObjects}(source, list, range)$

### 1.12.112 AddTraceMap (for IsCapCategory, IsFunction)

▷ AddTraceMap( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation TraceMap.  $F : (alpha) \mapsto \text{TraceMap}(alpha)$ .

### 1.12.113 AddCoRankMorphism (for IsCapCategory, IsFunction)

▷ AddCoRankMorphism( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation CoRankMorphism.  $F : (a) \mapsto \text{CoRankMorphism}(a)$ .

### 1.12.114 AddCoTraceMap (for IsCapCategory, IsFunction)

▷ AddCoTraceMap( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation CoTraceMap.  $F : (alpha) \mapsto \text{CoTraceMap}(alpha)$ .

### 1.12.115 AddCoclosedCoevaluationForCoDual (for IsCapCategory, IsFunction)

▷ AddCoclosedCoevaluationForCoDual( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation CoclosedCoevaluationForCoDual.  $F : (a) \mapsto \text{CoclosedCoevaluationForCoDual}(a)$ .

### 1.12.116 AddCoclosedCoevaluationForCoDualWithGivenTensorProduct (for IsCapCategory, IsFunction)

▷ AddCoclosedCoevaluationForCoDualWithGivenTensorProduct( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `CoclosedCoevaluationForCoDualWithGivenTensorProduct`.  
 $F : (s, a, r) \mapsto \text{CoclosedCoevaluationForCoDualWithGivenTensorProduct}(s, a, r)$ .

### 1.12.117 AddInternalCoHomTensorProductCompatibilityMorphismInverse (for IsCapCategory, IsFunction)

▷ `AddInternalCoHomTensorProductCompatibilityMorphismInverse(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `InternalCoHomTensorProductCompatibilityMorphismInverse`.  
 $F : (list) \mapsto \text{InternalCoHomTensorProductCompatibilityMorphismInverse}(list)$ .

### 1.12.118 AddInternalCoHomTensorProductCompatibilityMorphismInverseWithGivenObjects (for IsCapCategory, IsFunction)

▷ `AddInternalCoHomTensorProductCompatibilityMorphismInverseWithGivenObjects(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `InternalCoHomTensorProductCompatibilityMorphismInverseWithGivenObjects`.  
 $F : (source, list, range) \mapsto \text{InternalCoHomTensorProductCompatibilityMorphismInverseWithGivenObjects}(source, list, range)$ .

### 1.12.119 AddIsomorphismFromInternalCoHomToTensorProductWithCoDualObject (for IsCapCategory, IsFunction)

▷ `AddIsomorphismFromInternalCoHomToTensorProductWithCoDualObject(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `IsomorphismFromInternalCoHomToTensorProductWithCoDualObject`.  
 $F : (a, b) \mapsto \text{IsomorphismFromInternalCoHomToTensorProductWithCoDualObject}(a, b)$ .

### 1.12.120 AddIsomorphismFromTensorProductWithCoDualObjectToInternalCoHom (for IsCapCategory, IsFunction)

▷ `AddIsomorphismFromTensorProductWithCoDualObjectToInternalCoHom(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `IsomorphismFromTensorProductWithCoDualObjectToInternalCoHom`.  
 $F : (a, b) \mapsto \text{IsomorphismFromTensorProductWithCoDualObjectToInternalCoHom}(a, b)$ .

### 1.12.121 AddMorphismFromTensorProductToInternalCoHom (for IsCapCategory, IsFunction)

▷ AddMorphismFromTensorProductToInternalCoHom( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation MorphismFromTensorProductToInternalCoHom.  $F : (a, b) \mapsto \text{MorphismFromTensorProductToInternalCoHom}(a, b)$ .

### 1.12.122 AddMorphismFromTensorProductToInternalCoHomWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddMorphismFromTensorProductToInternalCoHomWithGivenObjects( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation MorphismFromTensorProductToInternalCoHomWithGivenObjects.  $F : (s, a, b, r) \mapsto \text{MorphismFromTensorProductToInternalCoHomWithGivenObjects}(s, a, b, r)$ .

### 1.12.123 AddMorphismToCoBidual (for IsCapCategory, IsFunction)

▷ AddMorphismToCoBidual( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation MorphismToCoBidual.  $F : (a) \mapsto \text{MorphismToCoBidual}(a)$ .

### 1.12.124 AddMorphismToCoBidualWithGivenCoBidual (for IsCapCategory, IsFunction)

▷ AddMorphismToCoBidualWithGivenCoBidual( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation MorphismToCoBidualWithGivenCoBidual.  $F : (a, r) \mapsto \text{MorphismToCoBidualWithGivenCoBidual}(a, r)$ .

## Chapter 2

# Examples and Tests

### 2.1 Test functions

#### 2.1.1 AdditiveMonoidalCategoriesTest

▷ `AdditiveMonoidalCategoriesTest(cat, a, L)` (function)

The arguments are

- a CAP category *cat*
- an object *a*
- a list *L* of objects

This function checks for every operation declared in `AdditiveMonoidalCategories.gd` if it is computable in the CAP category *cat*. If yes, then the operation is executed with the parameters given above and compared to the equivalent computation in the opposite category of *cat*. Pass the options

- `verbose := true` to output more information.
- `only_primitive_operations := true`, which is passed on to `Opposite()`, to only primitively install dual operations for primitively installed operations in *cat*. The advantage is, that more derivations might be tested. On the downside, this might test fewer `dual_pre/postprocessor_funcs`.

#### 2.1.2 BraidedMonoidalCategoriesTest

▷ `BraidedMonoidalCategoriesTest(cat, a, b)` (function)

The arguments are

- a CAP category *cat*
- objects *a, b*

This function checks for every operation declared in `BraidedMonoidalCategories.gd` if it is computable in the CAP category *cat*. If yes, then the operation is executed with the parameters given above and compared to the equivalent computation in the opposite category of *cat*. Pass the options

- `verbose := true` to output more information.
- `only_primitive_operations := true`, which is passed on to `Opposite()`, to only primitively install dual operations for primitively installed operations in `cat`. The advantage is, that more derivations might be tested. On the downside, this might test fewer `dual_pre/postprocessor_funcs`.

### 2.1.3 ClosedMonoidalCategoriesTest

▷ `ClosedMonoidalCategoriesTest(cat, a, b, c, d, alpha, beta, gamma, delta, epsilon, zeta)` (function)

The arguments are

- a CAP category `cat`
- objects `a, b, c, d`
- a morphism  $\alpha : a \rightarrow b$
- a morphism  $\beta : c \rightarrow d$
- a morphism  $\gamma : a \otimes b \rightarrow 1$
- a morphism  $\delta : c \otimes d \rightarrow 1$
- a morphism  $\varepsilon : 1 \rightarrow \text{Hom}(a, b)$
- a morphism  $\zeta : 1 \rightarrow \text{Hom}(c, d)$

This function checks for every operation declared in `ClosedMonoidalCategories.gd` if it is computable in the CAP category `cat`. If yes, then the operation is executed with the parameters given above and compared to the equivalent computation in the opposite category of `cat`. Pass the options

- `verbose := true` to output more information.
- `only_primitive_operations := true`, which is passed on to `Opposite()`, to only primitively install dual operations for primitively installed operations in `cat`. The advantage is, that more derivations might be tested. On the downside, this might test fewer `dual_pre/postprocessor_funcs`.

### 2.1.4 CocomonoidalCategoriesTest

▷ `CocomonoidalCategoriesTest(cat, a, b, c, d, alpha, beta, gamma, delta, epsilon, zeta)` (function)

The arguments are a CAP category `cat` objects `a, b, c, d`

- a morphism  $\alpha : a \rightarrow b$
- a morphism  $\beta : c \rightarrow d$
- a morphism  $\gamma : 1 \rightarrow a \otimes b$

- a morphism  $\delta : 1 \rightarrow c \otimes d$
- a morphism  $\varepsilon : \text{coHom}(a, b) \rightarrow 1$
- a morphism  $\zeta : \text{coHom}(c, d) \rightarrow 1$

This function checks for every operation declared in `CoclosedMonoidalCategories.gd` if it is computable in the CAP category  $cat$ . If yes, then the operation is executed with the parameters given above and compared to the equivalent computation in the opposite category of  $cat$ . Pass the options

- `verbose := true` to output more information.
- `only_primitive_operations := true`, which is passed on to `Opposite()`, to only primitively install dual operations for primitively installed operations in  $cat$ . The advantage is, that more derivations might be tested. On the downside, this might test fewer `dual_pre/postprocessor_funcs`.

### 2.1.5 MonoidalCategoriesTensorProductAndUnitTest

▷ `MonoidalCategoriesTensorProductAndUnitTest(cat, a, b)` (function)

The arguments are

- a CAP category  $cat$
- objects  $a, b$

This function checks for every operation declared in `MonoidalCategoriesTensorProductAndUnit.gd` if it is computable in the CAP category  $cat$ . If yes, then the operation is executed with the parameters given above and compared to the equivalent computation in the opposite category of  $cat$ . Pass the options

- `verbose := true` to output more information.
- `only_primitive_operations := true`, which is passed on to `Opposite()`, to only primitively install dual operations for primitively installed operations in  $cat$ . The advantage is, that more derivations might be tested. On the downside, this might test fewer `dual_pre/postprocessor_funcs`.

### 2.1.6 MonoidalCategoriesTest

▷ `MonoidalCategoriesTest(cat, a, b, c, alpha, beta)` (function)

The arguments are

- a CAP category  $cat$
- objects  $a, b, c$
- a morphism  $\alpha : a \rightarrow b$
- a morphism  $\beta : c \rightarrow d$

This function checks for every operation declared in `MonoidalCategories.gd` if it is computable in the CAP category *cat*. If yes, then the operation is executed with the parameters given above and compared to the equivalent computation in the opposite category of *cat*. Pass the options

- `verbose := true` to output more information.
- `only_primitive_operations := true`, which is passed on to `Opposite()`, to only primitively install dual operations for primitively installed operations in *cat*. The advantage is, that more derivations might be tested. On the downside, this might test fewer `dual_pre/postprocessor_funcs`.

### 2.1.7 RigidSymmetricClosedMonoidalCategoriesTest

▷ `RigidSymmetricClosedMonoidalCategoriesTest(cat, a, b, c, d, alpha)` (function)

The arguments are

- a CAP category *cat*
- objects *a, b, c, d*
- an endomorphism  $\alpha : a \rightarrow a$

This function checks for every object and morphism declared in `RigidSymmetricClosedMonoidalCategories.gd` if it is computable in the CAP category *cat*. If yes, then the operation is executed with the parameters given above and compared to the equivalent computation in the opposite category of *cat*. Pass the options

- `verbose := true` to output more information.
- `only_primitive_operations := true`, which is passed on to `Opposite()`, to only primitively install dual operations for primitively installed operations in *cat*. The advantage is, that more derivations might be tested. On the downside, this might test fewer `dual_pre/postprocessor_funcs`.

### 2.1.8 RigidSymmetricCoclosedMonoidalCategoriesTest

▷ `RigidSymmetricCoclosedMonoidalCategoriesTest(cat, a, b, c, d, alpha)` (function)

The arguments are

- a CAP category *cat*
- objects *a, b, c, d*
- an endomorphism  $\alpha : a \rightarrow a$

This function checks for every object and morphism declared in `RigidSymmetricCoclosedMonoidalCategories.gd` if it is computable in the CAP category *cat*. If yes, then the operation is executed with the parameters given above and compared to the equivalent computation in the opposite category of *cat*. Pass the options

- `verbose := true` to output more information.
- `only_primitive_operations := true`, which is passed on to `Opposite()`, to only primitively install dual operations for primitively installed operations in `cat`. The advantage is, that more derivations might be tested. On the downside, this might test fewer `dual_pre/postprocessor_funcs`.

## 2.2 Basics

Example

```

gap> LoadPackage( "MonoidalCategories" );
true
gap> vecspaces := CreateCapCategory( "VectorSpaces" );
VectorSpaces
gap> ReadPackage( "MonoidalCategories",
>               "examples/VectorSpacesMonoidalCategory.gi" );
true
gap> z := ZeroObject( vecspaces );
<A rational vector space of dimension 0>
gap> a := QVectorSpace( 1 );
<A rational vector space of dimension 1>
gap> b := QVectorSpace( 2 );
<A rational vector space of dimension 2>
gap> c := QVectorSpace( 3 );
<A rational vector space of dimension 3>
gap> alpha := VectorSpaceMorphism( a, [ [ 1, 0 ] ], b );
A rational vector space homomorphism with matrix:
[ [ 1, 0 ] ]
gap> beta := VectorSpaceMorphism( b,
>                                [ [ 1, 0, 0 ], [ 0, 1, 0 ] ], c );
A rational vector space homomorphism with matrix:
[ [ 1, 0, 0 ],
  [ 0, 1, 0 ] ]
gap> gamma := VectorSpaceMorphism( c,
>                                 [ [ 0, 1, 1 ], [ 1, 0, 1 ], [ 1, 1, 0 ] ], c );
A rational vector space homomorphism with matrix:
[ [ 0, 1, 1 ],
  [ 1, 0, 1 ],
  [ 1, 1, 0 ] ]
gap> IsCongruentForMorphisms(
>   TensorProductOnMorphisms( alpha, beta ),
>   TensorProductOnMorphisms( beta, alpha ) );
false
gap> IsOne( AssociatorRightToLeft( a, b, c ) );
true
gap> IsCongruentForMorphisms(
>   gamma, LambdaElimination( c, c, LambdaIntroduction( gamma ) ) );
true
gap> IsZero( TraceMap( gamma ) );
true
gap> IsCongruentForMorphisms(
>   RankMorphism( DirectSum( a, b ) ), RankMorphism( c ) );

```



```
true  
gap> IsOne( Braiding( b, c ) );  
false  
gap> IsOne( PreCompose( Braiding( b, c ), Braiding( c, b ) ) );  
true
```

## Chapter 3

# Code Generation for Monoidal Categories

### 3.1 Monoidal Categories

#### 3.1.1 WriteFileForMonoidalStructure

▷ `WriteFileForMonoidalStructure(key_val_rec, package_name, files_rec)` (function)

**Returns:** nothing

This function uses the dictionary `key_val_rec` to create a new monoidal structure. It generates the necessary files in the package `package_name` using the file-correspondence table `files_rec`. See the implementation for details.

### 3.2 Closed Monoidal Categories

#### 3.2.1 WriteFileForClosedMonoidalStructure

▷ `WriteFileForClosedMonoidalStructure(key_val_rec, package_name, files_rec)`  
(function)

**Returns:** nothing

This function uses the dictionary `key_val_rec` to create a new closed monoidal structure. It generates the necessary files in the package `package_name` using the file-correspondence table `files_rec`. See the implementation for details.

### 3.3 Coclosed Monoidal Categories

#### 3.3.1 WriteFileForCoclosedMonoidalStructure

▷ `WriteFileForCoclosedMonoidalStructure(key_val_rec, package_name, files_rec)`  
(function)

**Returns:** nothing

This function uses the dictionary `key_val_rec` to create a new coclosed monoidal structure. It generates the necessary files in the package `package_name` using the file-correspondence table `files_rec`. See the implementation for details.

## Chapter 4

# The terminal category with multiple objects

This is an example of a category which is created using `CategoryConstructor` out of no input.

This category “lies” in all doctrines and can hence be used (in conjunction with `LazyCategory`) in order to check the type-correctness of the various derived methods provided by `CAP` or any `CAP`-based package.

### 4.1 Constructors

## Chapter 5

# MonoidalCategories automatic generated documentation

### 5.1 MonoidalCategories automatic generated documentation of properties

#### 5.1.1 IsBraidedMonoidalCategory (for IsCapCategory)

▷ `IsBraidedMonoidalCategory( $C$ )` (property)

**Returns:** true or false

The property of the category  $C$  being braided monoidal.

#### 5.1.2 IsClosedMonoidalCategory (for IsCapCategory)

▷ `IsClosedMonoidalCategory( $C$ )` (property)

**Returns:** true or false

The property of the category  $C$  being closed monoidal.

#### 5.1.3 IsCoclosedMonoidalCategory (for IsCapCategory)

▷ `IsCoclosedMonoidalCategory( $C$ )` (property)

**Returns:** true or false

The property of the category  $C$  being coclosed monoidal.

#### 5.1.4 IsMonoidalCategory (for IsCapCategory)

▷ `IsMonoidalCategory( $C$ )` (property)

**Returns:** true or false

The property of the category  $C$  being monoidal.

#### 5.1.5 IsStrictMonoidalCategory (for IsCapCategory)

▷ `IsStrictMonoidalCategory( $C$ )` (property)

**Returns:** true or false

The property of the category  $C$  being strict monoidal.

### 5.1.6 IsRigidSymmetricClosedMonoidalCategory (for IsCapCategory)

▷ IsRigidSymmetricClosedMonoidalCategory( $C$ ) (property)

**Returns:** true or false

The property of the category  $C$  being rigid symmetric closed monoidal.

### 5.1.7 IsRigidSymmetricCoclosedMonoidalCategory (for IsCapCategory)

▷ IsRigidSymmetricCoclosedMonoidalCategory( $C$ ) (property)

**Returns:** true or false

The property of the category  $C$  being rigid symmetric coclosed monoidal.

### 5.1.8 IsSymmetricClosedMonoidalCategory (for IsCapCategory)

▷ IsSymmetricClosedMonoidalCategory( $C$ ) (property)

**Returns:** true or false

The property of the category  $C$  being symmetric closed monoidal.

### 5.1.9 IsSymmetricCoclosedMonoidalCategory (for IsCapCategory)

▷ IsSymmetricCoclosedMonoidalCategory( $C$ ) (property)

**Returns:** true or false

The property of the category  $C$  being symmetric coclosed monoidal.

### 5.1.10 IsSymmetricMonoidalCategory (for IsCapCategory)

▷ IsSymmetricMonoidalCategory( $C$ ) (property)

**Returns:** true or false

The property of the category  $C$  being symmetric monoidal.

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